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Theory of an X-Ray Interferometer in the Form of an Array of Planar Compound Refractive Lenses

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Abstract—The results of theoretical analysis of the interference pattern created by an X-ray multilens interferometer in the case of an arbitrary number of planar compound refractive lenses are presented. The full widths at half maximum of the resonance peaks in the transverse and longitudinal directions relative to the direction of synchrotron radiation are calculated at distances corresponding to the fractional Talbot effect. A relation between the widths is shown to exist that is very close to the width relation in the case of focusing by a single lens. A difference between the fractional and full Talbot effects is discussed, and the necessary conditions for the transverse and longitudinal coherence of radiation are analyzed, the satisfaction of which guarantees that undistorted interference peaks will be observed experimentally.

Keywords: X-ray interferometer, refractive lens, Talbot effect

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INTRODUCTION

The fabrication of X-ray focusing optical systems, i.e., compound refractive lenses, has been under intensive development since 1996 [1]. They have advantages for focusing X-ray radiation with a high photon energy (more than 30 keV) as compared with other systems. At present, two technologies for fabricating these lenses are most popular. The first one uses the pressing of material by parabolic punches to create a circular (2D) aperture [2]. The second one creates planar lenses for one-dimensional (1D) focusing. Modern planar compound refractive lenses are the surface layer of a crystal of specific thickness, which contains vertical steps confined by a parabolic profile. In fact, X-ray radiation is focused in the crystal's surface layer on convex cavities, unlike visible light, which focuses on the convex material. This is associated with the fact that a real part of the complex refractive index n of X-ray radiation is less than unity: $n = 1 - \delta + i\beta$. The β parameter is connected with the linear absorption factor by a simple relationship: $\mu = 2K\beta$, where $K = 2\pi/\lambda$ is the wavenumber, and λ is the wavelength of monochromatic X-ray radiation.

Typically a silicon crystal is used since the methods of microstructuring (electron-beam lithography and deep anisotropic etching) of silicon surfaces are very well developed [3–5]. These lenses have an extremely short focal distance (up to 1 cm) and make it possible to reduce a beam in the focus down to sizes ten times smaller than a micrometer, therefore they are called nanofocusing lenses. An important property of these lenses is also their small aperture (50 μm and less). Although, generally speaking, this is a drawback, but for third-generation sources of synchrotron radiation (e.g., ESRF in Grenoble, France), it is compensated for by the fact that the size of the beams themselves also is very small, while the size of the transverse (spatial) coherence is still less and commensurable with the effective aperture of these lenses.

On the other hand, the small aperture of planar compound refractive lenses made it possible to take the next step, namely, to create bilens [6] and multilens [7] interferometers, in which several planar compound refractive lenses are created on the same crystal in the direction perpendicular to that of beam propagation such that they are parallel to each other, touching the edges of the apertures. A scheme of the experiment with this interferometer is shown in Fig. 1. Although a single lens is shown schematically in the figure, a "train" of lenses with a large curvature radius of the parabolic surface is used in practice. If the focal distance of this lens is more than three times less than its length, then compound and single lenses with the same effective radius of curvature work in the same way [8]. The lenses collect an incident beam into two or more foci, after which the beams diverge as if the foci were secondary sources of X-ray radiation. At some distance along the axis of radiation propagation (z axis of a Cartesian coordinate system), the beams from different secondary sources overlap and interfere if the sum of apertures of the lenses (that create them) is less than the size of transverse coherence. In this case, all secondary sources are coherent. We will

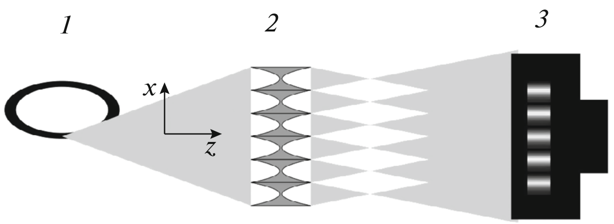

Fig. 1. Scheme of the experiment with the use of an X-ray multilens interferometer: (*1*) source of synchrotron radiation; (*2*) interferometer (a single lens is shown as the equivalent of a "train" of N lenses, the curvature radius of the parabolic surface of which is N times more); (*3*) detector (a fragment of the interference pattern is shown).

assume that this condition is satisfied, though, in reality, this is not always the case.

This work presents the results of theoretical analysis of the interference pattern created by an X-ray multilens interferometer in the case of an arbitrary number of lenses. Main attention is paid to the resonance structure that appears at distances of $z_N = d^2/\lambda N$ from the secondary sources, where d is the distance between secondary sources in the direction perpendicular to that of beam propagation (i.e., is equal to the lens aperture), and N is an integer. It is conventional to call the emergence of this structure the fractional Talbot effect, though between it and the Talbot effect (which consists in the fact that secondary sources reproduce themselves at distances of nz_T, where $z_T = 2d^2/\lambda$ and n is an integer), substantial differences exist, the demonstration of which is also the aim of this work. Additionally, the conditions for transverse and longitudinal coherence are discussed, which must be satisfied to see the interference pattern experimentally.

GENERAL FORMULAS AND THEIR COROLLARIES

Let us consider the most typical case when the length of the planar compound refractive lenses is much less than the distance to the focused image of the source. In this case, numerical calculation of the optical properties of the X-ray multilens interferometer can be carried out in the context of the conventional theory of X-ray phase contrast [9]. In this theory, an interferometer is described by the transmission function $T(x)$, depending on the coordinate x in the direction perpendicular to that of beam propagation (Fig. 1), namely:

$$T(x) = \exp\left(-iK[\delta - i\beta]t(x)\right), \tag{1}$$

where $t(x)$ is the varying thickness of the lens material along the beam parallel to the optical axis and passing through the point x.

Let the distance between the radiation source and the interferometer be z_0, and the distance between the interferometer and the detector be z_1. Then the distribution of the intensity $I(x)$ over the detector can be calculated by the formulas

$$I(x) = |a(x_0)|^2, \quad a(x_0) = \int dx_1 P(x_0 - x_1, Z)T(x_1), \tag{2}$$

where $x_0 = x\,(z_0/z_t)$, $Z = z_0 z_1/z_t$, $z_t = z_0 + z_1$, while the function $P(x, z)$ is the Fresnel propagator

$$P(x,z) = (i\lambda z)^{-1/2}\exp\left(i\pi\frac{x^2}{\lambda z}\right). \tag{3}$$

As it follows from (2), the interference pattern is the same for both parallel and divergent beams.

However the position of the pattern in space (z_1) and its transverse sizes (x) depend on the distance z_0. For a parallel beam ($z_0 = \infty$), we have $x = x_0$, $z_1 = Z$. For a divergent beam:

$$x = x_0\left(1 + \frac{z_1}{z_0}\right), \quad z_1 = \frac{Z}{1 - Z/z_0}. \tag{4}$$

Thus, we derive in the divergent beam the same pattern, but with an increased transverse size and at a distance farther from the interferometer. Formulas (4) should be taken into account in experimentally studying the optical properties of the multilens interferometer. However for theoretical analysis, it is sufficient to restrict oneself to the case of the parallel beam when $z_0 = \infty$.

Let the interferometer be an array of M lenses and its aperture be limited to a slit. Then the function $t(x)$ in (1) can be written as

$$t(x) = \sum_{k=1}^{M} t_k(x), \quad t_k(x) = \frac{(x - x_k)^2}{R}\theta(d - 2|x - x_k|). \tag{5}$$

Here d is the distance between the lens centers equal to their physical aperture, $R = R_0/N$ is the effective curvature radius of the parabolic profile of the lens on the condition that R_0 is the physical curvature radius for a single element, N is the number of elements, and $\theta(x)$ is the Heaviside function, equal to unity for a positive argument and to zero for a negative argument. The coordinates of the lens centers are equal to $x_k = d(k - [M + 1]/2)$. We will consider the symmetric case when M is an even number.

Formulas (1)–(5) have been used for the development of a computer program which allows one to take into account fine effects related to a limitation of the aperture of lenses due to absorption in the inhomogeneous material. However, general properties can be obtained without numerical calculations. The system of interferometer lenses focuses a parallel beam on points with transverse coordinates x_k. These foci can be considered as secondary sources located at a distance of f from the interferometer. Let us consider such distances $z_{1N} = f + z_N$ from the interferometer to the detector, passing which, the waves will have the same phase at the optical axis ($x = 0$) if the phase difference equal to 2π multiplied by an integer is neglected. It is obvious that at these distances, the relative intensity of the radiation will be M^2 times higher than that from a single lens.

The length of the path from the secondary source with the number k to the optical axis at the distance z is equal to

$$\begin{aligned} r_k &= (z^2 + x_k^2)^{1/2} \approx z + \frac{x_k^2}{2z} \\ &= z + \frac{d^2}{z}\frac{(k-(M+1)/2)^2}{2}. \end{aligned} \tag{6}$$

At the indicated distances z the difference in paths for different sources is equal to an integer number of wavelengths. This difference for sources with arbitrary numbers k and i is

$$r_k - r_i = r_{ki}^{(0)} = \frac{d^2}{z}\left[(k-i)\left(\frac{k+i}{2} - \frac{M+1}{2}\right)\right]. \tag{7}$$

It is easy to verify that the expression in square brackets is an integer. Indeed, k, i, and M are integers and M is an even number. If $k - i$ is the odd number, then $k + i$ is also the odd number. Consequently, the expression in parentheses is equal to the integer. If $k - i$ is an even number, then the expression in parentheses is a half-integer, and by multiplying it by an even number, we derive an integer again.

Consequently, the resonance condition can be written as $d^2/z_N = \lambda N$, where N is an arbitrary integer which we will call the order of resonance. From this condition, we derive the formula for distances $z_N = d^2/\lambda N$, at which resonance arises, i.e., all secondary sources interfere constructively.

One can also easily calculate the period of the interference pattern. For this purpose, it is necessary to repeat all calculations for a point with coordinate x on the detector, suggesting that $z = z_N$. We will give the calculation result straightaway:

$$r_k - r_i = r_{ki}^{(0)} - \lambda\frac{xN}{d}(k-i). \tag{8}$$

It follows from this formula that the period of the interference structure is $p = d/N$. In other words, the pattern at a distance, which is N times less than the main-resonance distance, has an N-times smaller period, while the period of the main resonance is equal to that of location of the sources.

The case of a bilens interferometer is a particular one. If $M = 2$, then the expression in square brackets of Eq. (7) is zero. Consequently, the resonance condition holds at any distance z. Accordingly, the period is dependent of the distance and is equal to $p = \lambda z/d$.

ANALYTICAL THEORY OF AN IDEAL INTERFEROMETER

Let us consider the structure of interference fringes at the resonance distances z_N. Formally, these may be very short distances for large N values. However, for the multilens interferometer with a large number of lenses M and a relatively small aperture of each lens, the beams from all lenses will not be able to interfere at small distances, since they will be separated in space. We will assume that the distances z_N are still large enough that all beams can overlap at least in a small region near the optical axis.

Let us consider an ideal interferometer, which focuses an incident plane wave on the secondary point sources, and let us measure the distance z from these sources. The wave function under the above described conditions is equal to the sum of Fresnel propagators:

$$\begin{gathered} E(x,z) = (i\lambda z)^{-1/2}\sum_{k=1}^{M}\exp\left(i\pi\frac{(x-x_k)^2}{\lambda z}\right), \\ x_k = d\left(k - \frac{M+1}{2}\right). \end{gathered} \tag{9}$$

The detector measures the radiation intensity $I(x, z)$ as a square of the modulus of this function. Accordingly, we derive

$$\begin{gathered} I(x,z) = \frac{1}{\lambda z}\left(M + 2\sum_{k=2}^{M}\sum_{j=1}^{k-1}\cos(\varphi_{kj}(x,z))\right), \\ \varphi_{kj} = \pi\frac{2x(x_j - x_k) + x_k^2 - x_j^2}{\lambda z}. \end{gathered} \tag{10}$$

Let us substitute the coordinate values from (9), and to calculate the double sum, we will regroup the terms, separating pairs of terms with the same distance between the sources. Namely, for each value of index k, we will summarize all indices $j = k - m$, where $m = 1, 2, \ldots, k - 1$. As a result, for the function $K(x, z) = \lambda z I(x, z)$, we derive

$$\begin{gathered} K(x,z) = M + 2\sum_{k=2}^{M}\sum_{m=1}^{k-1}\cos(\psi_{km}(x,z)), \\ \psi_{km} = \frac{\pi m}{\lambda z}\left(2xd + [M + m + 1 - 2k]d^2\right). \end{gathered} \tag{11}$$

We note that the term in the cosine argument ψ_{km}, which is independent of x, does not depend on index k. Again, let us regroup the terms using the substitution $k = (M + m + 1)/2 + l$ such that a sum over m would be the first one, and we finally derive the expression

$$\begin{aligned} &K(x,z) \\ &= M + 2\sum_{m=1}^{M-1}\sum_{l=-(M-m-1)/2}^{(M-m-1)/2}\cos\left(\frac{2\pi m}{\lambda z}(xd - ld^2)\right). \end{aligned} \tag{12}$$

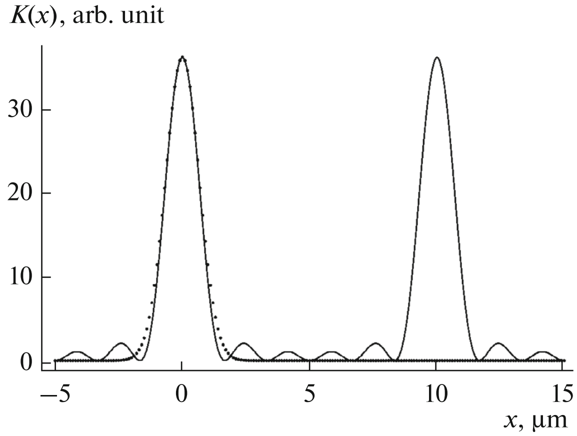

Fig. 2. Function $K(x)$ for $M = 6$, $p = 10$ µm (solid curve) and its approximation by a Gaussian function (dots).

In this case, M is an even number, while m is an integer; consequently, the index l may be both an integer and a half-integer number.

If $M = 2$, then we derive only one term with $m = 1$, $l = 0$. With an increase in the number of lenses M, the number of terms in Eq. (12) grows rapidly. So, for $M = 6$, we have 15 terms, divided into five groups with different periods. It is important to note that the product ml is always an integer, even for half-integer values of l. It is easy to understand that the period of the function $K(x)$, and, consequently, also the period of the interference structure is equal to $p = \lambda z/d$; but higher harmonics exist with a smaller period. However, at the resonance distances $z = z_N = d^2/\lambda N$, the argument ld^2 does not influence the result, since it is equal to 2π multiplied by an integer. At these distances, we derive the simpler expression

$$K(x, z_N) = M + 2\sum_{m=1}^{M-1}(M-m)\cos\left(2\pi\frac{m}{p}x\right), \quad p = \frac{\lambda z_N}{d} = \frac{d}{N}. \tag{13}$$

It is interesting that the period of the interference structure is independent of the number of lenses M, but the height of the maxima depends on it. With $x = 0$, we have $K(0) = M^2$. On the other hand,

$$\langle K\rangle = \int_0^p dx K(x, z_N) = M. \tag{14}$$

Consequently, the peak value is M times higher than the mean, which is M times higher than the value for a single lens. It is evident that the mean is equal to the intensity for the incoherent source.

A more complicated issue is determination of the peak width in the directions both perpendicular and parallel to the direction of beam propagation. The numerical calculations show that the peak width in the perpendicular direction is approximately M times less than the period, and the peak is described roughly by a Gaussian function. Let us suggest that for large M values and near the point $x = 0$, the function $K(x) \approx M^2\exp(-\alpha x^2)$. The coefficient α can be determined by comparing the second term of the expansion of the exponential into power series in x^2 with the expansion of exact expression (13). Then the peak's full width at half maximum (FWHM) $w_t = 1.665\alpha^{-1/2}$ can be derived from the Gaussian function. The first terms of the expansion are

$$K(x, z_N) = M + 2\sum_{m=1}^{M-1}(M-m)\left(1-\frac{b^2m^2}{2}\right) = M^2 - M^2\frac{M^2-1}{12}b^2. \tag{15}$$

Here $b = 2\pi x/p$, while exact values of the sums can be taken from the tables or the Internet [10]. Thus, we derive

$$\alpha = \left(\frac{2\pi}{p}\right)^2\frac{M^2-1}{12}, \quad w_t = \frac{1.665}{\alpha^{1/2}} = p\frac{0.9180}{(M^2-1)^{1/2}}. \tag{16}$$

It is interesting that for $M = 2$, Eq. (16) yields $w_t = 0.52p$, whereas for large M values, we derive $w_t = 0.92p/M$. In Fig. 2, the exact function $K(x)$ for $M = 6$ and $p = 10$ µm is shown with a solid line, while its approximation by the Gaussian function is shown with dots. It can be seen that approximation by the Gaussian function works well.

Let us consider the longitudinal shape of the resonance peak. We assume in Eq. (12) that $x = 0$ and perform the substitution $z = (d^2/\lambda N)(1 + s/N)$. Only small values of s for which the approximation $1/z = (\lambda N/d^2)(1 - s/N)$ can be used will be considered. As a result, we derive the function

$$K_1(s) = M + 2\sum_{m=1}^{M-1}\sum_{l=-(M-m-1)/2}^{(M-m-1)/2}\cos(2\pi mls). \tag{17}$$

Now we can apply to this function the same method as above. So, the first two terms of the expansion into power series in s^2 are

$$K_1(0, s) \approx M^2 - (2\pi s)^2 C, \tag{18}$$

where

$$C = \sum_{m=1}^{M-1} m^2 \sum_{l=-(M-m-1)/2}^{(M-m-1)/2} l^2 = \frac{M^2(M^2-1)(M^2-4)}{720}. \tag{19}$$

The coefficient C has a simple physical meaning, though to calculate the sum directly is rather difficult. Indeed, the coefficient should be zero for $M = 0$, 1,

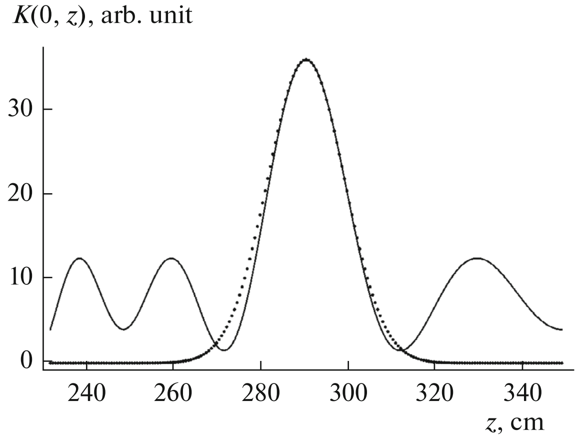

Fig. 3. Function $K(0, z)$ for $M = 6$, $E = 12$ keV, $d = 30$ μm, $N = 3$ (solid curve) and its approximation by a Gaussian function (dots).

and 2, since for these M values, the resonance structure is lacking. Exact calculation yields the correct value of the denominator.

Again, let us approximate the exact function (17) by the Gaussian function $M^2\exp(-\alpha s^2)$ near the resonance peak and derive the FWHM of function (17) in the form

$$w_s = \frac{1.665}{\alpha^{1/2}} = \frac{7.111}{[(M^2-1)(M^2-4)]^{1/2}}, \quad \alpha = (2\pi)^2\frac{(M^2-1)(M^2-4)}{720}. \tag{20}$$

The parameter w_s is versatile, but the real FWHM of the intensity along the optical axis with large M values is approximately equal to

$$w_l = \frac{d^2 w_s}{\lambda N^2} \approx \frac{7.111 p^2}{\lambda M^2} \approx 8.438\frac{w_t^2}{\lambda}. \tag{21}$$

This formula is less versatile, since it depends on λ. Figure 3 shows the exact calculated curve of the function $K(0,z)$ near the resonance peak for $M = 6$, $E = 12$ keV, $d = 30$ μm, and $N = 3$. It can be seen that the resonance peak is only three times higher as compared with other maxima and its FWHM is very close to that of the Gaussian curve.

It should be noted that the actual aperture of the lenses was not taken into consideration in these calculations. Nevertheless, the derived relationship between the transverse and longitudinal FWHM of the resonance peak is very similar to the analogous relation for focusing by a single compound refractive lens. For example, if the lens aperture is determined by absorption, then from the theory of focusing by a single lens [11], we have for the transverse FWHM of intensity at the focus $w_t = 0.6643(\lambda f\gamma)^{1/2}$, where $f = R/2\delta$ is the focal distance, $\gamma = \beta/\delta$. On the other hand, for the longitudinal FWHM, the estimate $w_l = (12)^{1/2} f\gamma = 7.850\, w_t^2/\lambda$ can be derived. The only difference between this formula and Eq. (21) is that the numerical factor is less by 7%.

TALBOT EFFECT FOR A FINITE NUMBER OF SOURCES

The Talbot effect [12] may be formulated as a full reproduction of the transverse periodic wave function with a period of d for radiation propagation along the optical axis (the z axis) over the distance $z_T = 2d^2/\lambda$. The effect can be proven easily using the Fourier transform. The periodic wave function can be presented in the form of the Fourier series:

$$E(x,0) = \sum_{m=-\infty}^{\infty} F(h_m)\exp(ih_m x), \quad h_m = \frac{2\pi}{d}m, \tag{22}$$

where

$$F(h_m) = \frac{1}{d}\int_{-d/2}^{d/2} dx E(x,0)\exp(-ih_m x). \tag{23}$$

The propagation of radiation over the distance z is described by convolution of the wave function with the Fresnel propagator:

$$E(x,z) = \int dx_1 P(x - x_1, z)E(x_1, 0). \tag{24}$$

Substituting (22) to (24) and taking the integral, we derive

$$E(x,z) = \sum_{m=-\infty}^{\infty} F(h_m)\exp\left(ih_m x - i\pi m^2\frac{\lambda z}{d^2}\right). \tag{25}$$

From this formula it follows right away that $E(x, z_T) = E(x, 0)$. In other words, not only the radiation intensity but also the complex wave function is repeated completely during propagation over the Talbot distance z_T.

This derivation is very simple for a fully periodic wave field. However, no conclusions can be drawn from it relative to a finite periodic system of point sources. The intensity distribution for a finite number of sources is described by Eq. (12), but at the Talbot distances, peaks arise at the points $x_k = d[k - (M - 1)/2] = \pm d/2, \pm 3d/2, \ldots$ instead of the points $0, \pm d, \pm 2d, \ldots$, as in the case of first-order resonance. This is the first distinction of the Talbot effect from resonances (the fractional Talbot effect).

Let us substitute into Eq. (12) the coordinates $x_T = \pm d/2$ and $z_T = 2d^2/\lambda$ and calculate the sums:

$$K(x_T, z_T) = M + 2\sum_{m=1}^{M-1}\sum_{l=-(M-m-1)/2}^{(M-m-1)/2}\cos\left(\pi\left[\pm\frac{m}{2} + ml\right]\right) = \frac{M^2}{2}. \quad (26)$$

From this formula it follows that the intensity maximum for Talbot peaks in the case of a finite number of sources M is half as high, as the one in the case of resonances. This is the second distinction of Talbot peaks from resonances. Since the average intensity is maintained, it is evident that the transverse FWHM of Talbot peaks is nearly twice as large as the one for the resonance peaks. More detailed analysis of the Talbot effect is beyond the scope of this work, since Talbot peaks are at a relatively large distance, and it is hard to observe them experimentally.

NECESSARY CONDITIONS FOR TRANSVERSE CONERENCE

The transverse coherence of radiation incident on the object is directly related to the angular size of the source determined at the distance from the source to the object. If the coordinate x_s of the point source describes its deviation from the main optical axis, then a temporary optical axis can be considered, on which both the source and the object are located. It is obvious that this axis will be turned at the angle $\alpha = x_s/z_0$ around the central point of the object. The interference pattern for this source will be roughly the same, only for the detector will it be shifted from the main optical axis by the distance $x_s' = \alpha z_1 = x_s z_1/z_0$. As a result, the complete pattern for the entire source will correspond to the averaged pattern for the point source with the interval of averaging equal to the size of the source projection $s = Sz_1/z_0$, where S is the actual size of the source.

In order to see peaks of Nth resonance without distortions, the source projection should be half the transverse width of the resonance. With allowance for Eqs. (4) and (16), we have

$$2S\frac{z_1}{z_0} < 0.9\frac{d}{MN}\left(1 + \frac{z_1}{z_0}\right), \quad z_1 = \frac{Z}{1 - Z/z_0}, \quad Z = f + \frac{d^2}{\lambda N}. \quad (27)$$

Typically, the focal distance of a compound refractive lens is much less than the distances to the resonances, which are formed by all lenses, and we can neglect it. Combining the written formulas, we can rewrite the condition in the form

$$dM < L_{tc}, \quad L_{tc} = 0.45\frac{\lambda z_0}{S}. \quad (28)$$

This condition has a very simple physical meaning. To observe experimentally the calculated interference FWHM corresponding to the interference of all secondary sources, the total aperture of the interferometer dM should be less than the length of the transverse coherence L_{tc}, as was determined in [13, 14]. If condition (28) is not satisfied for all interferometer lenses, then the fringe pattern will correspond to an interferometer with a smaller number of lenses; moreover, the number of lenses is determined just from condition (28).

However, condition (28) has no practical use, since it does not allow one to simulate experimental curves of interference. For this purpose, it is more useful to calculate the interference pattern for a point source in the form of the intensity distribution function and then to calculate the convolution of this function with that of the source-projection brightness, for which the FWHM is equal to $s = Sz_1/z_0$, where S is the FWHM of the source-brightness distribution. Typically, approximation of the function of the source brightness by the Gaussian function is accepted. If the real size of the source is unknown in the experiment, then this procedure allows one to determine it from the comparison of calculated curves with experimental ones.

NECESSARY CONDITIONS FOR LONGITUDINAL COHERENCE

The notion of longitudinal coherence is introduced for nonmonochromatic radiation, whose photon energy is distributed within a certain range ΔE. In accordance with the uncertainty principle, pulses of this radiation have a finite lifetime $\tau = \hbar/(2\Delta E)$, where $\hbar$ is Planck's constant divided by 2π. If the measurement time substantially exceeds the radiation-pulse duration, then the phase relations for waves with different frequencies, i.e., different energies, are lost, and it is necessary to calculate the convolution of the function of radiation intensity for different monochromatic harmonics with the function describing the radiation spectrum. In other words, function (12) should be averaged over the radiation wavelength $\lambda = hc/E$, where h is Planck's constant, c is the speed of light, the constant $hc = 1.24$ nm keV. Since function (12) is the double sum of cosines, then each cosine should be averaged independently.

In the case of a synchrotron-radiation source, the real spectrum is determined by the monochromator. Let us consider a simple situation when the spectrum is described approximately by the Gaussian function

$$W(E, \Delta E) = \frac{1}{(2\pi)^{1/2}\sigma}\exp\left(-\frac{E^2}{2\sigma^2}\right), \quad \sigma = \frac{\Delta E}{2.355}. \quad (29)$$

Then for the average of cosine, we derive the formula:

$$\langle\cos(aE)\rangle = \frac{1}{(2\pi)^{1/2}\sigma}\int dE_1\cos(a[E + E_1])\exp\left(-\frac{E_1^2}{2\sigma^2}\right), \quad (30)$$

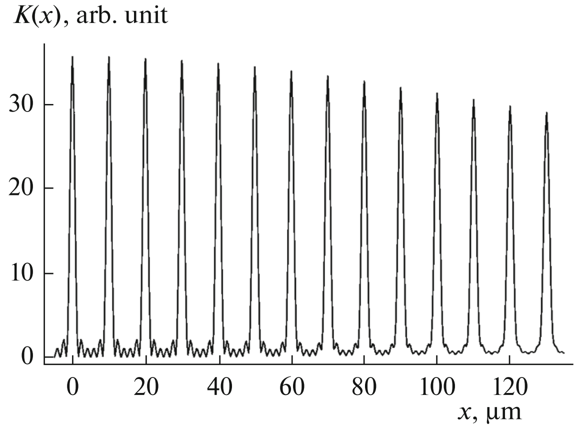

Fig. 4. Function $K(x)$ for $M = 6$, $E = 12$ keV, $p = 10$ μm, and $\Delta E = 0.1$ keV.

where $a = 2\pi md(x - ld)/(hcz)$. The integral is taken analytically, and as a result of calculation, we derive that each cosine within the sums over m and l in Eq. (12) is additionally multiplied by the factor

$$C_{ml} = \exp\left(-3.56\frac{L_{ml}^2}{L_{lc}^2}\right), \quad L_{ml} = \frac{md(x - ld)}{z}, \quad L_{lc} = \frac{\lambda^2}{\Delta\lambda} = \frac{hc}{\Delta E}. \tag{31}$$

The parameter L_{lc} is known as the length of longitudinal coherence. Let us note that the quantity L_{ml} determines the path difference of beams for a pair of sources with a center at the point ld. When the coordinate x corresponds to the center of the pair, the path difference for it equals zero.

From this it follows that for a particular point of the detector with the coordinate x, the maximum contribution to the intensity will be made by the sources, which have the minimum path difference. It is evident that the number of such pairs is maximum at $x = 0$ and reduces as $|x|$ increases. Figure 4 shows the result of numerical calculation of the interference pattern with the following parameters: $E = 12$ keV, $M = 6$, $N = 3$, $d = 30$ μm, $\Delta E = 0.1$ keV. It can be seen that the central peaks did not change even in the case of a relatively high degree of nonmonochromaticity. For the resonance distances $z_N = d^2/\lambda N$, the ratio L_{ml}/L_{lc} can be written in another form, namely:

$$\frac{L_{ml}}{L_{lc}} = Nm\left(\frac{x}{d} - l\right)\frac{\Delta E}{E}. \tag{32}$$

From this formula it can be seen that the ratio $E/\Delta E$ is directly proportional to the number of peaks in the central part of the interference pattern, which can be seen experimentally.

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