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NANOPHOTONICS

On the Possibility of the Nanoscale Focusing of Synchrotron Radiation Using an Adiabatic Refractive Lens

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Abstract—A new version of the analytical theory for the nanoscale focusing of synchrotron radiation (SR) using compound refractive lenses (CRLs) is developed. A rather simple (from the point of view of both technical realization and theoretical calculations) model of an adiabatic refractive lens (ARL) in the form of a cascade of CRLs is proposed. It is shown that planar ARLs, which can be potentially implemented using silicon-surface microstructuring, can focus a SR beam to a size smaller than that, obtained in the use of CRLs ($w_c = \lambda(8\delta)^{-1/2}$, where λ is the wavelength, $\delta = 1 - n$, and n is the refractive index). It is found that, in the case of high-energy photons (more than 50 keV), the theoretically possible sizes of a focused beam of hard SR are less than 10 and 7 nm for silicon and nickel ARLs, respectively.

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INTRODUCTION

Focusing of beams of synchrotron radiation (SR) and X-ray free-electron lasers to sizes from hundreds to dozens and even units of nanometers is important for the development of nanotechnologies, because it opens new possibilities for the structural analysis of microobjects and nanoobjects due to a higher radiation intensity. Currently, the use of refracting X-ray optics (specifically, compound refractive lenses (CRLs)) is a promising direction in this field. Although CRLs became available to physicists only recently [1], they are widely used in SR sources of the third generation and technologies of their production are being actively developed.

The analytical theory of SR focusing using CRLs was formulated in [2, 3] and recently developed in [4], where it was shown that, to decrease the SR-beam size at the focus, one must increase the photon energy E and use CRLs with a small aperture A. An increase in E and a decrease in A lead to a reduction in the role of absorption and increase the efficiency of radiation refraction in CRLs. As a result, absorption ceases to affect the focusing process and the beam size in a CRL becomes determined only by its aperture. It was found that, under these conditions, the beam size at the focus is independent of both E and A and becomes approximately equal to the critical size $w_c = \lambda(8\delta)^{-1/2}$, where λ is the radiation wavelength and δ is the decrement of refractive index of the CRL material.

The critical size was introduced for the first time into the theory of X-ray optics in [5] as the minimum beam size, which can be achieved using focusing techniques. For example, the w_c values for silicon and nickel CRLs are 20 and 10 nm, respectively. At the same time, the model of an adiabatic refractive lens (ARL), which can make it possible to overcome this limit, was proposed and theoretically considered in [6]. In the ARL model [6], the refraction efficiency is increased using elements with gradually changing apertures depending on the beam compression in the ARL. It was shown that these lenses can theoretically focus beams up to 2 nm in size (however, only at unrealistic values of the parameters of the terminal ARL elements). A smooth change in the apertures of the ARL elements with the required accuracy is technologically impossible and, to date, no lenses with sufficient efficiency have been obtained. The theory of these lenses also requires further development.

The purpose of this study is to develop the analytical theory of an alternative ARL model, the apertures of elements in which change stepwise by some specific value at beam compression in the ARL. These ARLs are simpler from the point of view of both their preparation using planar technologies [7] and theoretical calculation of their focusing properties. A theory for the proposed ARL model is presented and the algorithm for calculating the focusing parameters is discussed. The results of calculating the minimum transverse sizes of the SR beam at the focus in the case of using elements, which are either being currently fabri-

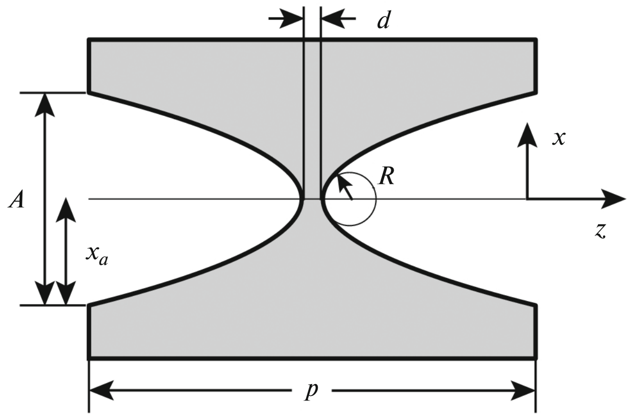

Fig. 1. Elementary lens (a CRL element) with the following parameters: A is the aperture, $x_a = A/2$, R is the parabolic-surface curvature radius, d is the thickness of the thin part (bridge), and p is the element length along the optical axis.

cated or can be theoretically manufactured, are also discussed.

THEORY OF ARL FOCUSING

An ARL is a cascade of several CRLs. Each CRL consists of identical elementary lenses (elements), the parameters of which are shown in Fig. 1. The main parameters are aperture A, curvature radius R, and thin-part thickness d. The element length $p = d + x_a^2/R$, $x_a = A/2$. It was shown in the analytical theory of CRLs [2–4] that if the wave function of the radiation, incident on the CRL, is

$$\psi_0(x) = C_0 \exp(-i\pi x^2/\lambda f_{0c}),\quad 1/f_{0c} = 1/f_0 - i\lambda C_w^2/w_0^2, \tag{1}$$

where $C_w = (2\ln 2/\pi)^{1/2} = 0.6643$ and w_0 is the full width at half maximum (FWHM) of the Gaussian function $|\psi_0(x)|^2$, the wave function of radiation after CRL has the same form with C_0 and f_{0c} replaced by C and f_c, respectively. New and old values of parameters are interrelated according to the following recurrence formulas:

$$f_c = a_L/b_L,\quad C = C_0(f_{0c}/a_L)^{1/2}, \tag{2}$$

where

$$a_L = f_{0c}c_L - z_c s_L,\quad b_L = c_L + s_L f_{0c}/z_c, \tag{3}$$

$$c_L = \cos(L/z_c),\quad s_L = \sin(L/z_c),\quad z_c = (pR/2\eta)^{1/2}. \tag{4}$$

Here, $L = pn_1$ is the CRL length, n_1 is the number of elements in the CRL, $\eta = 1 - n = \delta - i\beta$, and β is the absorption index (imaginary part of the refractive index n).

In the cascade ARL model, the CRL aperture decreases stepwise by some step S upon compression of the SR beam during its propagation in an ARL. The number of elements n_1 in the CRL is chosen so that the SR-beam width decreases due to the refraction of rays upon their propagation in the CRL to the size of the new aperture. In this case, the aperture of the elements of the following CRL in the cascade equals the beam size at the end of the previous CRL. Together with aperture $A = 2x_a$, it is reasonable to vary the curvature radius R. It can also be decreased according to some law (e.g., $R = x_a/4$). Otherwise, the elements will be very short, which is technologically impractical. With regards to parameter d, one tends to minimize it; however, it can hardly be decreased to a greater extent due to technical limitations.

In the analytical theory of CRLs, aperture is not taken into account. The intensity (i.e., squared modulus of the wave function) always has the shape of a Gaussian curve with the FWHM w_0. It was shown in [4] using an accurate iterative calculation taking into account the aperture that the beam at the CRL output has a limited width, which coincides with the path of rays from the aperture edge. Within this width, the accurate solution coincides with that, obtained using analytical theory, even if the effective aperture, determined by absorption, exceeds the real aperture. The path of refracted rays can also be calculated independently using another method. To this end, one can use the laws of geometric optics. Because refraction at one surface of CRL elements is very small, it is sufficient to use a simplified version of geometric optics, which corresponds to the approximation of analytical theory (specifically, one can assume that parameter δ depends only on x and is independent of z). This approximation was justified in [4].

To calculate the number of elements n_1, required to compress the SR beam by the step of a change in the aperture S, one must consider the ray path, corresponding to the edge of the aperture of this CRL (i.e., with coordinate $x = x_a$). According to [4], the ray-path coordinate at the end of a CRL of length L is described by the following expression:

$$x(L) = x_a C_L + \theta_a L_c S_L, \tag{5}$$

where

$$C_L = \cos(L/L_c),\quad S_L = \sin(L/L_c),\quad L_c = (pR/2\delta)^{1/2}. \tag{6}$$

Here, θ_a is the tilt angle of the ray at the aperture edge with respect to the optical axis (axis z in Fig. 1) before the CRL. The parameters in (6) correspond to those in (4) at zero absorption. A change in the ray tilt angle can be calculated according to formula

$$\theta(L) = dx(L)/dL = \theta_a C_L - x_a S_L/L_c. \tag{7}$$

Usually, in SR sources, the distance from the source to the experimental station is quite long and one can approximately assume that the initial ray angle θ_a is zero at small transverse-beam sizes. An iterative increase in the number of elements n_1 by unity can make it possible to find the value, at which condition $x(L) < x_a - s$ $(s = S/2)$ is satisfied for the first time. This value determines the length of the first CRL, which is written in a table. At this length, we determine $\theta(L)$ and replace parameters x_a and θ_a with new values and pass to the second CRL. This procedure must be repeated until the last CRL in the cascade is calculated. The number of elements in the last CRL can be calculated in the same way using the previously set value of the minimum beam size at the ARL end.

When the lengths of all CRLs in the cascade are found, formulas (1)–(4) make it possible to calculate a change in the wave function of radiation after transmission through all CRLs and to obtain parameter f_c at the ARL end. As was noted above, the initial conditions can be assumed the same as for a plane incident wave (i.e., $f_{0c}^{-1} = 0$, $C_0 = 1$). Then, according to (1), we obtain at the ARL end

$$f = (\mathrm{Re}(f_c^{-1}))^{-1}, \quad w_e = C_w[-\lambda/\mathrm{Im}(f_c^{-1})]^{1/2}. \tag{8}$$

We note that analytical CRL theory does not take into account the aperture and is valid only when absorption is high and the FWHM of the Gaussian curve of the intensity at the ARL end is smaller than the CRL aperture.

However, it is not the case at weak absorption and the real beam size at the ARL end is limited by its aperture and refraction. Therefore, the beam size at the focus must be calculated taking into account the beam width A_e at the ARL end, which is determined using geometric optics (i.e., beyond analytical theory). This question was analyzed in detail in [4], where it was shown that the size and shape of the SR-intensity distribution at the focus are determined by the function $[F(u_0)]^2$, where

$$F(u_0) = \frac{1}{u_0}\int_0^{u_0} du\cos(u)\exp(-gu^2/u_0^2). \tag{9}$$

Here, the following parameters are introduced:

$$u_0 = (\pi A_e/\lambda f)x, \quad g = (\ln 2/2)(A_e/w_e)^2. \tag{10}$$

We note that parameter g is expressed explicitly in terms of the ratio of the beam size at the ARL end according to geometric optics and the Gaussian FWHM w_e of the beam according to analytical theory.

The FWHM of function $[F(u_0)]^2$, which is denoted as $w_u(g)$, determines the SR-beam size at the ARL focus as

$$w_f = (\lambda f/\pi A_e)w_u(g). \tag{11}$$

The analytical solution for $w_u(g)$ can be obtained only in limiting cases. For example, $w_u(0) = 2.783$ at zero absorption. In another limiting case ($g \gg 1$), squared function (9) becomes a Gaussian function with the FWHM $w_u(g) = (8\ln 2)^{1/2}g^{1/2} = 2.355g^{1/2}$. A universal solution for all g values can be found using numerical methods. In particular, it was revealed that function $w_u(g)$ barely differs from the linear dependence $w_u(g) = w_u(0) + 0.498g$ in the range of $0 \le g \le 6$ and the $w_u(g)$ value at $g > 6$ coincides with high accuracy with the above-described analytical estimate. Thus, by calculating parameters A_e, w_e, and f at the ARL end, one can determine the FWHM of the SR-intensity curve at the focus taking into account the aperture.

RESULTS AND DISCUSSION

It is most interesting to estimate the SR-beam size at the ARL focus using the parameters of already existing planar lenses. The measurement results, obtained upon the use of planar CRLs on the surface of silicon with apertures of 30 and 50 μm, were reported in [8, 9]. CRLs with apertures of 10 μm were also fabricated; however, the corresponding results have not been published yet. In principle, existing technology makes it possible to obtain CRLs with apertures of less than 10 μm. It was noted in [4] that CRLs from heavier elements (e.g., nickel) can be promising for achieving limiting nanofocusing. Nickel CRLs with apertures of about several micrometers have not been developed yet; however, in principle, it is also possible. For example, there exist planar kinoform lenses from nickel with characteristic sizes of a minimum segment of 2.6 μm [10].

In the aforementioned planar silicon CRLs, the ratio of R and A is $R = A/8$. At the same time $p = d + 2A$. The bridge thickness is usually $d = 2$ μm. In calculations, the same R/A ratio was used and $d = 2$ μm for elements with an aperture of up to 10 μm and $d = 1$ μm for elements with smaller apertures. The step S of the decrease in the aperture depended on the number of steps n_s. The number of elements of the last stage was calculated using the SR-beam size at the ARL end, which was set equal to 0.5 μm. Calculations were carried out using a program, written in the ACL programming language [11], which is executed by a program, written in the Java language.

Obviously, some ARL parameters (e.g., the number of elements in each CRL of the cascade) depend on the photon energy E. Figure 2 shows the results of calculating the path of the ray, beginning from the edge of the aperture A_0 of the first CRL in the cascade at different numbers of stages n_s. Calculations were performed for $E = 50$ keV. The doubled height of the curves shows the geometric size of the SR beam in a silicon ARL with a starting aperture of $A_1 = 50$ μm. The curves for $n_s = 1$, 3, 7, and 25 are shown in Fig. 2 (the numbers are indicated). We note that the steps of a decrease in the aperture are set to be $S = 0$, 20, 8, and 2 μm, respectively. The apertures of the last CRL in the cascade are $A_n = 50$, 10, 2 and 2 μm, respectively.

The curve for $n_s = 1$ corresponds to a conventional CRL. It can be seen that the angle between the ray path and optical axis becomes almost constant,

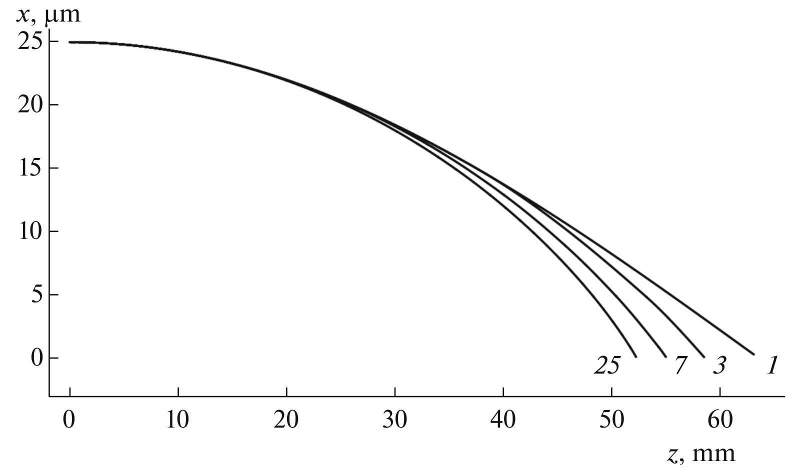

Fig. 2. Plots of ray paths, corresponding to the edge of the starting ARL aperture (Si, $A = 50$ μm, $E = 50$ keV) at the number of stages 1, 3, 7, and 25. The numbers of stages are indicated at the curve ends. The step in a decrease in the aperture equals 0, 20, 8, and 2 μm, respectively. The path curvature increases at an increase in the number of stages.

approximately beginning with the lens midpoint. Thus, a long CRL focuses radiation inefficiently, because of linear beam convergence in empty space. As a result, the aforementioned focusing limit is obtained at a rather small aperture and radiation wavelength; i.e., the beam size at the focus is 22.5 nm.

With an increase in the number of steps, the ray paths turn down strongly and, after the ARLs, the angle between the curves and the argument axis increases. It means that the width of the SR angular spectrum at the focus increases, which, according to the general law of optics (an analog of the coordinate—momentum uncertainty principle in quantum mechanics), leads to a decrease in the focused-beam size.

The results of calculating the main focusing parameters for energy $E = 50$ keV and different versions of silicon ARLs are listed in Table 1. In these calculations, step $S = A_1/n_s$. It follows from the reported data that the beam size at the focus for the ARL with a starting aperture of $A_1 = 50$ μm and number of steps $n_s = 50$ decreases by a factor of more than 2 in comparison with the conventional CRL and equals 9.7 nm.

Table 1. ARL focusing parameters (Si, $E = 50$ keV)

A_1, μm	n_s	f, mm	g	w_f, nm
50	1	0.313	1.38	22.5
	5	0.223	1.83	13.8
	25	0.170	2.08	10.4
	50	0.159	2.12	9.7
30	1	0.372	0.83	20.7
	3	0.267	1.01	14.2
	15	0.190	1.20	10.1
	30	0.174	1.24	9.4

We note that, as evidenced by an increase in parameter g in Eq. (9), absorption in the ARL material increases at an increase in the number of steps. This fact is explained as follows: at an increase in the number of steps, the ray paths in the ARL pass through a thicker part of the elements. Obviously, the presence of absorption decreases the beam size at the focus, because it decreases the angular SR width.

To achieve larger beam compression, one must add lenses with smaller apertures; however, to date, elements with apertures of less than 1 μm cannot be fabricated. In addition, one can theoretically decrease the influence of absorption by decreasing the thickness of the bridges between elements (this solution is also technologically impossible). At the same time, absorption can be reduced by decreasing the starting aperture and, therefore, the total length of ARL. It follows from the data in Table 1 that, indeed, the role of absorption is reduced for the ARL with the starting aperture $A_1 = 30$ μm and the same step of a change in the aperture as in the first case; however, the beam size at the focus barely changes and equals 9.4 nm for a lens with the number of steps $n_s = 30$. Thus, absorption barely affects the focusing result for silicon ARLs with any starting aperture at $E = 50$ keV and the main factor, which determines the beam size at the focus, is the size of the aperture of the last stage.

The results of calculating the main focusing parameters for the energy $E = 50$ keV for nickel ARLs, are listed in Table 2. It was found that, in this case, absorption is high ($g > 6$) even for a starting aperture of $A_1 = 30$ μm and the beam size at the focus is larger than that for the same ARL made of silicon. Obviously, to decrease the role of absorption, one must decrease the starting aperture to a greater extent and use higher photon energies.

The calculation results for an energy of $E = 80$ keV and silicon and nickel ARLs with the step of a change in the aperture $S = 1$ μm are listed in Table 3. It can be

Table 2. ARL focusing parameters (Ni, $E = 50$ keV)

A_1, μm	n_s	f, mm	g	w_f, nm
50	1	0.230	13.2	36.9
	5	0.136	19.8	22.8
	25	0.097	23.1	17.6
	50	0.088	23.7	16.5
30	1	0.184	8.9	26.8
	3	0.132	11.5	19.3
	15	0.103	14.1	14.3
	30	0.093	14.6	13.4

Table 3. ARL focusing parameters (Si, Ni, $E = 80$ keV)

A_1, μm	n_s	f, mm	g	w_f, nm
		Si		
50	50	0.255	1.77	9.3
30	30	0.276	1.04	9.1
		Ni		
30	30	0.144	6.85	8.9
10	10	0.182	2.11	6.9

seen that the focused-beam size for silicon ARLs decreases only slightly at an increase in the photon energy and all still exceeds 9 nm.

For the nickel ARL with a starting aperture of $A_1 = 30$ μm, parameter g still exceeds 6 and the beam size at the focus is about 9 nm. At a decrease in the starting aperture to 10 μm, $g < 6$ and, correspondingly, absorption ceases to affect the focusing result. As a result, the minimum beam size at the focus is also determined by the aperture of the last lens and, in this case, equals 6.9 nm. Thus, the nickel ARL focuses radiation most efficiently at a small starting aperture and high photon energy $E > 80$ keV.

It is known that planar CRLs with a large focal length are also used successfully for two-dimensional focusing (to this end, two CRLs in different planes (x, z) and (y, z) are used). We note that the first CRL should have a longer focal length and the step should be no less than the length of the second CRL. However, an ARL with a large number of CRLs is almost ideally adiabatic and focuses the beam very closely to its end. Therefore, only the second ARL can provide the minimum focus size in the case of using two planar ARLs in the crossed geometry.

CONCLUSIONS

The developed analytical theory of focusing using the cascade model of an adiabatic refractive lens makes it possible to calculate relatively simply the main parameters of the SR beam at the focus. This procedure is reduced to iterative calculation of the beam width and wave-function parameters at the ARL end and relatively simple integration for obtaining the curve of the focused-beam intensity.

At reasonable values of the parameters, these lenses can be fabricated using planar technologies. We note that the minimum focused-beam size is determined mainly by the aperture of the last cascade stage.

At the current stage of technology development, silicon and nickel planar ARLs make it possible to compress the SR beam to sizes of less than 10 and 7 nm, respectively. For silicon ARLs, this value can be obtained at a radiation energy of $E = 50$ keV. In the case of nickel ARLs, one has to use higher energies $E = 80$ keV.

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CONFLICT OF INTEREST

We declare that we have no conflict of interest.

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