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Computer Simulation of the Effect of Focusing X Rays by Means of Refractive–Diffractive Lens

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Abstract—The features of focusing X rays using a refractive–diffractive lens (RDL), which is a system of two asymmetrically reflecting crystals with asymmetry factors whose product is equal to unity, and a refractive lens with a large focal length, are theoretically studied. Crystals make it possible to shorten the focal length of the lens by b^2 times, where b is the asymmetry factor of the second crystal. A detailed numerical simulation of the effect of radiation focusing using the RDL has been performed. The universal computer program XRWP was used, which was created to calculate the effects of coherent X-ray optics. Analytical formulas are obtained for the optimal aperture and radius of curvature of the lens, as well as for the width of the radiation spectrum that can be focused.

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INTRODUCTION

The X-ray refractive index in a substance differs very little from unity. In addition, there are no transparent materials for X rays. For this reason, attempts to focus an X-ray beam using the refraction effect were unsuccessful for hundred years after discovering X rays. This problem was solved in 1996 using a compound refractive lens (CRL) [1]. CRLs have been widely used to focus narrow beams on third-generation synchrotron radiation (SR) sources, especially after designing planar and parabolic lenses [2]. It turned out to be fortunate that the real part of X-ray refractive index for all materials is smaller than unity, i.e., $n_r = 1 - \delta$. In this case, a focusing lens should have a concave rather than convex surface profile, due to which absorption losses are significantly reduced.

For example, a biconcave beryllium lens with a curvature radius $R = 1$ mm at the parabola apex focuses a plane wave at a distance $F_1 = R/2\delta = 448$ m for the photon energy $E = 17.48$ keV, corresponding to the $K_{\alpha 1}$ line in the spectrum of X-ray tube molybdenum anode. The problem was solved using the on-line program [3]. At an aperture $A = 2$ mm the length of this lens is $p = A^2/4R = 1$ mm, disregarding the bridge thickness, which cannot be more than 1% of the lens length. If 400 such lenses are put together, the focal length will shorten by a factor of 400 to the value $F_{400} = 1.12$ m, which is quite acceptable for experiments even in a medium-size laboratory, using a source with small transverse sizes.

Since the total length of this CRL exceeds 40 cm, the thin-lens approximation is inaccurate for it, and a more complex theory [4] must be used. In addition, due to the absorption on aperture edges, the beam will in fact be focused on an effective aperture A_e of smaller size. In this case, the calculation within the on-line-program [3] gives the following result: $F_{400} = 1.18$ m (counting from the middle of the CRL length) and $A_e = 0.473$ mm. Despite the fact that the aperture is reduced by a factor of 4, it is still fairly large. The beam size in the focus for a point source is $w_f = 0.084$ μm, whereas the maximum relative intensity is $I_m/I_0 = 5296$. Here, I_0 is the radiation intensity before the CRL and I_m is the maximum intensity in the CRL focus.

Another way to reduce the focal length of a lens consisting of one element was considered in [5]. It was proposed to reflect asymmetrically the X-ray beam from the atomic planes of silicon crystals installed before the lens and after it. This lens, based on the combined effect of refraction in the lens and diffraction in the crystals, was called a refractive–diffractive lens (RDL). Crystals were cut in the form of plates so as to provide a relatively large angle between the plate surface and the atomic planes reflecting the beam. The crystal orientation is such that the beam reflected from the first crystal is expanded, i.e., its width is divided by the coefficient $b_1 = 1/b$. After the reflection from the second crystal, the beam is compressed, i.e., its width is divided by the coefficient $b_2 = b$.

The parameter $b = \sin(\theta_0)/\sin(\theta_h) >> 1$ is equal to the asymmetry factor for the second crystal, it greatly exceeds unity. Here, b_k ($k = 1, 2$) is the asymmetry parameter for a specific crystal and θ_0 and θ_h are the

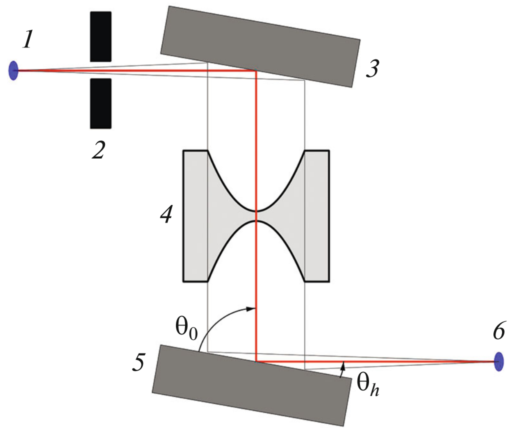

Fig. 1. Schematic of the experiment: (*1*) point source; (*2*) slit limiting the beam; (*3*) first crystal, expanding the beam, (*4*) lens focusing the beam, (*5*) second crystal, compressing the beam, and (*6*) focus, i.e., the point where the beam becomes a secondary source.

angles between the crystal surface and the directions of the incident and reflected beams, respectively. The schematics of a possible experiment is illustrated in Fig. 1, which shows the angles θ_0 and θ_h, the point source (*1*); the slit limiting the beam (*2*); the first crystal, expanding the beam (*3*); the lens focusing the beam (4); the second crystal, compressing the beam (*5*); and the focus, i.e., the point at which the beam becomes a secondary source (*6*). The distances between objects *1* and *2*, as well as between *5* and *6*, are much larger in a real experiment, they are reduced for compactness in the figure. Other distances are small and can be neglected in the calculation. In this scheme the first crystal is used to reduce the distance from the source to the lens, while the second crystal reduces the lens focal length. The divergence of electromagnetic radiation is known to be determined by its first Fresnel zone, the diameter of which for a spherical wave is $2(\lambda z)^{1/2}$, where $\lambda = hc/E$ is the radiation wavelength, h is Planck's constant, c is the speed of light, and z is the distance from the object to the observation point. Therefore, when a beam is compressed by a factor of b, the focal length is reduced by a factor of b^2. This fact, noted a year before [5], was published and used for another purpose in [6].

With allowance for the aforementioned fact, having chosen $b = 20$, one can obtain a focal length of 1.12 m for one lens (indicated above) at reflection from the second crystal; in this case, it is not necessary to put together 400 lenses. This method appears attractive, taking into account the high cost of refractive lenses for focusing SR. The first crystal with inverse asymmetry is also necessary to make the focus position in the transverse plane independent of the radiation wavelength. The first crystal in [6] reflected symmetrically, and the reflection by the second crystal was used to separate different SR wavelengths in space.

A numerical calculation of the parameters of RDL-focused X-ray beam was performed in [5] based on the analytical formulas for the radiation intensity in the focus, which were derived using the crystal and lens propagators directly in real space. These formulas are relatively complicated; the calculations were performed in only few points and with low accuracy. Another approach to the numerical simulation of X-ray optics effects is developed and implemented in the universal computer program XRWP [7], which uses the modular principle of wave optics. The essence of this principle is that a change in the wave function (WF) in the plane oriented perpendicular to the beam direction is taken into account on a system points in a specified computational grid. A change in WF is successively recalculated when passing through each object and each distance from one object to another.

Having a set of calculation modules for each object and involving empty space in the calculation, one can calculate any experimental scheme, including RDL. When the calculation is reduced to convolution of two functions, the Fourier transform from real to reciprocal space and vice versa is used. The calculation is performed according to the fast Fourier transform (FFT) algorithm [8], which sharply shortens the calculation time. Another advantage of this method for solving X-ray optics problems is that the FFT algorithm needs a grid with many points and a small step, which makes it possible to describe easily intensity oscillations with an arbitrarily small period, generally occurring in coherent optics.

The purpose of this study was to perform a numerical simulation of the effect of RDL-aided SR focusing within the XRWP program and analyze theoretically the drawbacks of this focusing technique, related to the fact that the diffractive reflection of an SR beam by a crystal is implemented in a very narrow angular range. Analytical formulas are obtained for the optimal values of the aperture and refractive-lens curvature radius, as well as the width of the spectrum that can be focused by RDL. This analysis was not performed in [5]. At the same time, the calculation by the new method, the results of which are on the whole consistent with those obtained in [5], is performed more thoroughly.

STATEMENT OF THE PROBLEM AND METHOD FOR ITS SOLUTION

The main calculation formulas used in the XRWP program were described in [9]. Two-wave diffraction from the atomic lattice in a crystal is implemented in the (x, z) plane, the z axis is chosen along the beam direction, and the x axis is perpendicular to z. We restrict ourselves to the consideration of a one-dimensional lens, focusing radiation in the same plane. Let $F(x)$ be the WF of monochromatic radiation with a specified photon energy E at some point on the beam path, i.e., at a fixed value of coordinate z.

The WF transfer in empty space by some distance $z = z_2 - z_1$, where $z_{1,2}$ are the initial and final distances in empty space, is calculated in the form of a convolution $F(x)$ with a Fresnel propagator, which is a mathematical analog of the Huygens–Fresnel principle.

The convolution is calculated in three stages. The first stage implies calculation of the WF Fourier transform $F(q)$, where q is the coordinate in the reciprocal space, related to x. In the second stage, this function is multiplied by the Fourier transform of the Fresnel propagator

$$P(q,z) = \exp(-i(\lambda z/4\pi)q^2). \qquad (1)$$

The inverse Fourier transform is calculated in the third stage. This method was used in many author's publications. Its another advantage is the possibility of obtaining rapidly a two-dimensional array in the (x, z) plane for the SR intensity distribution in empty space. If WF is known at some distance z_1, it is not necessary to recalculate the entire experimental scheme in order to calculate the WF value at the distance z_2. It is sufficient to calculate only the convolution.

The WF transmission through a thin biconcave focusing lens can be taken into account via multiplying WF by the lens transmission function $T(x) = \exp(-i\pi x^2/\lambda F_1)$, if $|x| < A/2$. An SR beam is generally limited by a slit; in this case its width is equal to the RDL aperture, and $T(x) = 0$ beyond the aforementioned range. The accuracy of this approach is quite sufficient for a single lens. The XRWP program can calculate CRLs by more complex methods [10], which are applicable for an arbitrary number of lenses.

The asymmetric reflection by a single crystal is taken into account by calculating the convolution with the crystal propagator but in a more complex way. Here, the Fourier transform of the crystal propagator is a solution to the problem of plane wave diffraction from an atomic lattice, which was thoroughly considered a long time ago in textbooks [11, 12]. The program implements the most general case of asymmetric reflection from a multilayer crystal, using recurrence formulas. The recurrence formulas were derived in the general form in [13]. In [14] the calculation formulas for a multilayer crystal are presented in the most convenient form but only for the symmetric case. A generalization to the asymmetric case was performed in [6].

Below we give the formulas for calculating radiation in the reflected beam for a crystal of finite thickness in the asymmetric case of diffraction:

$$F(x) = \exp(-iq_0'xb_k)F'(xb_k), \qquad (2)$$

$$F'(x) = \int (dq/2\pi)\exp(iqx)P(q,z)P_c(q-q_0,b)F_0(q). \qquad (3)$$

Here, it is assumed that the beam WF $F_0(x)$ at a distance z_1 from the crystal is known, and it is necessary to find the WF at a distance z_2 after the reflection from the crystal. Correspondingly, the inverse Fourier transform is calculated from the product of the WF Fourier transform by the Fourier transform of the Fresnel propagator for the distance $z = z_1 + z_2 b_k^2$ and by the Fourier transform of the crystal propagator $P_c(q - q_0, b_k)$. In this case, we have [14]

$$P_c(q,b_k) = (R_1 - R_2 C\exp(i\phi))/(1 - C\exp(i\phi)), \qquad (4)$$

$$R_{1,2} = (\sigma \pm a)/sf, \quad a = (\sigma^2 - bfs^2)^{1/2}, \quad C = R_1/R_2, \quad \phi = at/\sin\theta_0, \qquad (5)$$

$$\sigma = qb_k\sin(2\theta_B) - i\mu_0(1+b)/2, \quad s = K\chi_h, \quad f = \chi_{-h}/\chi_h. \qquad (6)$$

Here, t is the crystal thickness, $K = 2\pi/\lambda$, χ_h and χ_{-h} are the complex diffraction parameters, $\mu_0 = K\chi_{0i}$ is the linear absorption coefficient, and χ_{0i} is the imaginary part of the complex parameter χ_0.

At the same time, the crystal responds strongly to a deviation from the Bragg condition, both when rotated at some angle φ and when the photon energy shifts (by ΔE) from the value corresponding to the Bragg condition. In this case [6],

$$q_0 = q_a + q_b, \quad q_a = -K\varphi, \quad q_b = K(\Delta E/E)\tan(\theta_B), \qquad (7)$$

$$q_0' = q_a(1 + 1/b_k) + q_b(1 - 1/b_k). \qquad (8)$$

It follows from (3) that the Fourier integral is first calculated on a standard grid of points. Then the point grid spacing for the calculation result should be divided by the parameter b_k according to formula (2). If $b_k > 1$, the spacing is reduced. A shift of the crystal angular position by φ and the photon energy shift ΔE from their values corresponding to the Bragg condition lead both to a change in reflection and to the occurrence of an additional phase factor, due to which the reflected beam deviates from its direction. It follows from (8) that, at $b_k = 1$, the reflected beam direction is independent of the photon energy shift. A similar situation occurs in the case of reflection in two crystals, when $b_1 b_2 = 1$.

CALCULATION RESULTS AND THEIR ANALYSIS

Figure 2 presents the results of calculating the X-ray intensity distribution in empty space after RDL focusing, which was described above and considered in [5]. The results were obtained for the following parameters: beryllium lens with $R = 1$ mm; $A = 2$ mm; silicon crystals; and asymmetric reflection 220 with asymmetry factors b_k, equal to $1/b$ for the first crystal and $b = 20$ for the second crystal. The RDL focuses the monochromatic radiation from a point source with a photon energy $E = 17.48$ keV. The diffraction parameters for the crystals were calculated using the on-line-program [15]. The focal length of this lens for plane waves, with allowance for the reduction caused by the

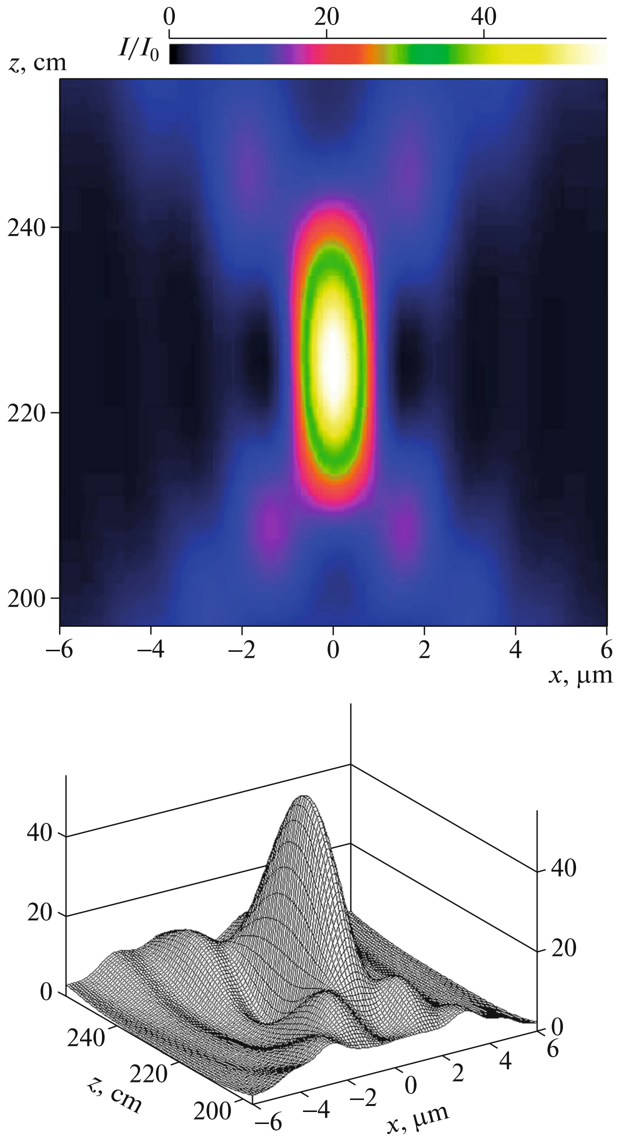

Fig. 2. Distribution of relative radiation intensity in empty space in the region near the point of RDL focusing. The photon energy is 17.58 keV; the distances from the RDL to the source and to the focus are $z_0 = z_1 = 2.24$ m; silicon crystals; 220 reflection; asymmetry factors $b = 1/20$ and 20 for first and second crystals, respectively. Beryllium lens, $R = 1$ mm, $A = 2$ mm.

reflection from the second crystal, is approximately $F_s = R/(2\delta b^2) = 1.12$ m. The real distance z_1 from the RDL to the focus depends on the distance z_0 from the source to the RDL. All three distances satisfy the lens formula: $z_0^{-1} + z_1^{-1} = F_s^{-1}$.

Figure 2 presents the case where $z_0 = z_1$. Correspondingly, $z_1 = 2F_s$, and focusing occurs at a distance of about 224 cm. The plot in Fig. 2 is a two-dimensional intensity distribution in the (x, z) plane; the coordinate z changes in the range corresponding to the lens focal depth. The intensity normalization was performed with allowance for the energy conservation law, i.e., the intensity in the focus is assigned by the intensity before the RDL. The relative intensity before the RDL is unity. The total intensity falling in the focus is equal, correspondingly, to the RDL aperture. If the RDL does not absorb, the total intensity in the focus is compressed into a peak, the integral from which is approximately equal to the product of the beam FWHM by the maximum intensity. Roughly speaking, the number of times the beam is compressed is equal to the number of times its maximum intensity increases. If this does not hold true, there are intensity losses.

The upper part of Fig. 2 shows colored intensity distribution with a low accuracy, but with indication of exact values of coordinates x and z. In the lower part of the figure the intensity distribution is shown in the axonometric surface projection in three-dimensional space. One can see well how intensity changes, but all changes are distorted by projecting. Figure 2 demonstrates that the intensity distribution is not purely Gaussian: it has complex tails with additional low maxima. The main peak has an almost standard shape, observed in the case of focusing by a refractive lens with a finite aperture and without absorption. It can be seen that the RDL focuses radiation, but, in comparison with the beam parameters in the focus for the CRL composed of 400 elements, the effect is rather modest.

Indeed, the peak half-width (full width at half maximum) in the focus is $w_f = 1.5$ μm and the height is $h_f = 51$. At these parameters the integral of the Gaussian is $S = 1.065 w_f h_f = 81$ μm, whereas the effective RDL aperture with allowance for the reflection by the crystal is 2 mm$/b$ = 100 μm. In other words, there are still small losses. Note that the results of the calculation performed in [5] show the same peak half-width, but the maximum is larger by a factor of 4. The reason for this difference is as follows: the value reported in [5] is the ratio of the focused intensity at the RDL focal point to the intensity in the same point without focusing, i.e., at a distance double as large as that in this study, while the dependence of intensity on distance in the absence of RDL is quadratic. Specifically this circumstance led to the effective increase in intensity by a factor of 4. Thus, the result of the calculation performed in [5] in quite a different way and with a very low accuracy, coincides with the calculation result obtained here.

It should be noted that the RDL is an analog of CRL only in the sense that it has the same focal length. The peak half-width in the focus for CRL, according to the data of the on-line-program [3], is 0.161 μm, and the peak height is 2904. In other words, the CRL focusing efficiency is many times higher. The reason for this difference is that the effective CRL aperture under these conditions is 510 μm. Being formed by absorption, it compresses the beam stronger than the

RDL by a factor of not 5 but almost 10. The specificity of focusing by a lens with absorption was discussed in [4]. Note that the maximum of relative intensity for CRL is larger by a factor of more than 50 than that for RDL. It is of interest that even if the first crystal would reflect the beam symmetrically the RDL anyway could not function normally. There are two reasons for this.

The first reason is that the focus position at asymmetric reflection depends strongly on the photon energy. In this case the lens demonstrates the emission spectrum with a high accuracy. This version was proposed in [6] as a new-type spectrometer. In this case the distance from the RDL to the source should be very large to make the angular width of the aperture small; otherwise, even symmetric reflection would not work. Let us consider the limiting case, where a plane wave is incident on the lens, i.e., $z_0 >> z_1$. Then $z_2 = F_s$. Let $F_0(x)$ determine the radiation WF directly behind the second crystal. The radiation is focused at the distance z_2 (the point x_0) and the WF in the focus, $F_1(x)$, is determined by the integral

$$F_1(x) = (i\lambda z_2)^{-1}\int dx_1 \exp(i(\pi/\lambda z_2)(x - x_1)^2)F_0(x_1). \quad (9)$$

According to (2), at a small shift of photon energy ΔE, the function $F_0(x)$ gains an additional phase factor $\exp(-iq_0'bx)$. The other changes in the function are small and can be neglected. Adding an exponential to the integrand, one can easily find that the focal point is shifted by

$$\Delta x_0 = -z_2 q_0' b/K = -z_2(\Delta E/E)\tan(\theta_B)(b-1). \quad (10)$$

With the parameters considered here, the shift of the focus by 1 μm is obtained at $\Delta E/E = 2.5 \times 10^{-7}$. Such a strong sensitivity to the spectrum can be suppressed in only the experimental scheme, when the product of the asymmetry parameters in the two RDL crystals is unity.

The second reason is that the angular range of asymmetric reflection of the second crystal is fairly narrow, and the crystal would not be able to reflect the whole angular aperture of the lens even for a lens of large focal length. Figure 3 shows the intensity distribution near the focus for the same RDL but at $z_0 =$ 50 m (a distance typical of third-generation SR sources). In this case, according to the lens formula, the beam is focused at $z_1 =$ 1.15 m. Since the focal length decreased, the peak half-width in the focus became smaller: $w_f = 0.898$ μm. It is of interest that the peak slightly shifted from the center (by 0.5 μm). The inequality of distances breaks symmetry, and the SR beam reflection by the crystals is not symmetric; therefore, a small shift occurs at reflection. The relative intensity in the maximum is $h_f = 61.5$.

It is of interest that the total intensity in this case is about $S = 1.065 w_f h_f = 55$ μm, i.e., the beam intensity focused by the lens is lost almost by half. The reason is

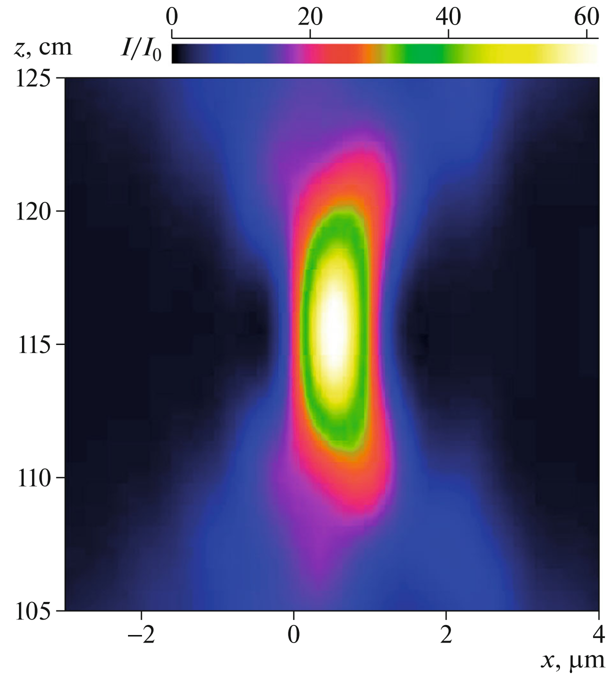

Fig. 3. Distribution of relative radiation intensity in empty space in the region near the point of RDL focusing at the same parameters as in Fig. 2, except for z_0 = 50 m and z_1 = 1.15 m.

that the shorter focal length increased the angular aperture of the lens, while the angular width of the reflection by the second crystal did not change; i.e., the second crystal reflected only part of intensity. The angular width Δ_θ for the beam incident on the second crystal (the input beam) is determined by the range of angles $\theta = q/K$ in which the parameter a in equation (5) is purely imaginary at zero absorption. Carrying out calculations, we obtain

$$\Delta_\theta = 2|\chi_h|/(b^{1/2}\sin(2\theta_B)). \quad (11)$$

At the values of parameters considered here, $\Delta_\theta = 2.4 \times 10^{-6}$ rad. At the same time, the angular aperture of the lens under consideration for plane waves is $A_\theta = A/F_1 = 4.5 \times 10^{-6}$ rad. In other words, the crystal reflected only half of the lens angular aperture.

When divergent radiation is incident on a lens, its angular aperture decreases, and the reflection by the second crystal becomes more complete. However, incomplete reflection of the beam by the first crystal may occur in this case. For the first crystal, the input-beam angular width is larger by a factor of $b = 20$, but the lens angular width is determined by the real distance to the source (z_0) after reducing the lens aperture by a factor of b.

Obviously, at specified crystal parameters, reflection indices, photon energy, and lens material, one cannot arbitrarily choose such lens parameters as R and A. For laboratory experiments, the case with

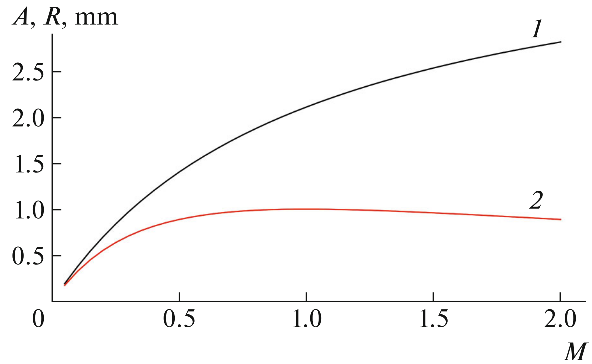

Fig. 4. Dependences of the optimal values of (*1*) aperture A and (*2*) surface curvature radius R of the lens on the parameter $M = z_1/z_0$ at the fixed distance $z_t = z_0 + z_1 =$ 4.48 m. All other parameters are the same as in Fig. 2.

a small distance from the source to the focus, $z_t = z_0 + z_1$, is most optimal if the detector is installed at the focal point. Proceeding from the condition $\Delta_\theta = A_\theta$ and the lens formula, the equations for calculating the aperture and curvature radius can be written as

$$A = 2|\chi_h| z_2 b^{3/2}/\sin(2\theta_B), \quad R = 2\delta z_2 b^2/(1+M), \quad M = z_2/z_1. \tag{12}$$

These equations make it possible to obtain the dependence of the parameters A and R on M. Here, A determines the integral relative intensity as $A_e = A/b$, and the formula $w_f = \lambda z_2/A_e$ allows one to estimate the beam size in the focus. Estimation is made over lens for its real focusing distance $z_2 b^2$ and with allowance for the beam compression by a factor of b. At the same time, the formula is standard if one uses the experimental distance after the second crystal and the effective lens aperture before the first crystal.

It follows from the presented equations that the beam size in the focus is independent of M at optimal parameters, because the optimal effective aperture linearly depends on the distance z_2. The beam size in the focus depends on the lens angular aperture, which does not change. Figure 4 shows the dependences of A and R on M at $z_t = 4.48$ m. The values at $M = 1$ are close to the data presented in Fig. 2, i.e., $A = 2$ mm, $R = 1$ mm. It can be seen that the total intensity increases with an increase in M. However, one should remember that the projection of source sizes also increases with M, which may increase significantly the beam size in the focus. In addition, lenses with a large aperture can work efficiently only for radiation of very high coherence, which is always problematic in laboratory experiments.

It is known that the diffractive reflection by crystals exists in only a limited spectral range of radiation. One can install crystals at correct angular positions, but one should not form unreasonably a monochromatic beam. Any radiation has a spectrum. Broadband radiation cannot be focused by a refractive lens, but the CRL sensitivity to the spectral width is not very high yet. The sensitivity of diffractive reflection is much higher. One can compensate the transverse shift of a focused beam but cannot obtain reflection when the Bragg condition is not satisfied.

According to (3), when the radiation photon energy deviates by ΔE from the value exactly corresponding to the Bragg condition for a fixed angular orientation of crystal, the angular range of the reflection shifts by $\Delta\theta_0 = q_b/K = (\Delta E/E)\tan(\theta_B)$. On the other hand, the angular range of the lens focusing does not change. If crystals are correctly oriented, it corresponds to the angular range of reflection by the second crystal. The angular range of reflection by the first crystal may be larger, and its shift will change nothing. However, after the reflection by the first crystal, the beam direction changes by the angle $\Delta\theta_1 = -\Delta\theta_0(b_1 - 1)$. The shift of the angular range relative to the beam direction is $\Delta\theta_2 = \Delta\theta_0 - \Delta\theta_1 = \Delta\theta_0/b$. It is known that a lens does not change the general beam direction, corresponding to the ray passing through the lens center; therefore, the lens can be disregarded in this consideration. Obviously, at $\Delta\theta_2 = C\Delta_\theta$, the beam intensity reflected by the crystal will decrease by half. A rough estimation of the parameter C gives $(1-2^{-1/2}) =$ 0.3. Practice has shown that a better coincidence with the calculation result is obtained at $C = 0.36$.

This condition gives an estimated width of radiation spectrum that can be focused by RDL in the form

$$(\Delta E/E)_{fwhm} = 1.44 b^{1/2}|\chi_h|/(\sin(2\theta_B)\tan(\theta_B)), \tag{13}$$

where the factor 1.44 is equal to $4C$. This estimate is in no way related to focusing and determined by only the crystals. For the case considered in Fig. 2, $(\Delta E/E)_{fwhm} = 1.8 \times 10^{-4}$, which is in good agreement with the results of numerical calculations according to the XRWP program, as well as with the calculation results reported in [5].

The dependence of the relative intensity integrated over coordinate x on the photon energy for the RDL under consideration, at the same parameters as in Fig. 2, is shown in Fig. 5. Note that this dependence is not quite symmetric, and its maximum is slightly shifted from the center. However, the deviations from the symmetric curve are small. It is of interest that focusing occurs for practically all energies, and only the maximum intensity changes. The decrease in the intensity is due to the fact that the part of the lens aperture that focuses rays decreases. The other part either does not fall on the lens or is not reflected by the second crystal.

Summing up, the following conclusion can be formulated: it is fairly difficult to increase the intensity by a factor of more than 100 in the experimental scheme considered here, and the beam width in the focus cannot be much smaller than 1 μm. These limitations are determined by the small angular width of asymmetric

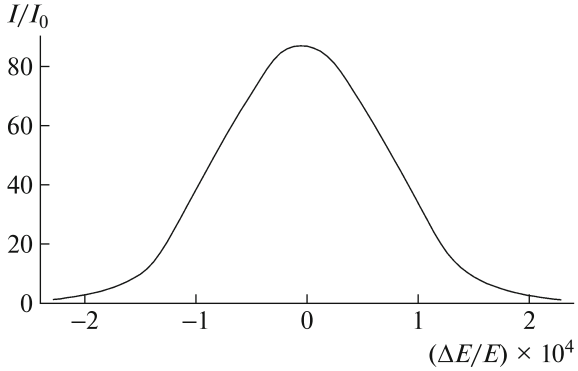

Fig. 5. Energy spectrum of the radiation focused by RDL, i.e., the intensity in the focus, integrated over the coordinate x, in dependence of the photon energy for the same RDL parameters as in Fig. 2.

reflection by a crystal with a decrease in the beam width.

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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