ISSN 2635-1676, Nanobiotechnology Reports, 2025, Vol. 20, No. 2, pp. 238–244. © Pleiades Publishing, Ltd., 2025.
Russian Text © The Author(s), 2025, published in Rossiiskie Nanotekhnologii, 2025, Vol. 20, No. 00000.

NANOPHOTONICS

New Version of the Theory of the Nanofocusing of Hard Synchrotron-Radiation Beams by a Long Compound Refractive Lens

V. G. Kohn[a],*

[a]*National Research Center "Kurchatov Institute," Moscow, Russia*

**E-mail: kohnvict@yandex.ru*

Received December 11, 2024; revised December 11, 2024; accepted March 5, 2025

Abstract—A new version of the theory of the nanofocusing of synchrotron radiation (SR) into nanometer transverse size using a long compound refractive lens (CRL) is developed, which adequately takes into account the aperture over the entire length of the CRL within the framework of geometrical optics, fully compatible with the theory based on the CRL propagator. For hard SR, absorption no longer completely limits the CRL aperture and taking into account the real aperture is important to obtain reliable information about the size of the SR beam at the focus. The new theory is almost analytical and allows one to determine the rocking curve of a long CRL with good accuracy, and also gives an answer in those exotic cases where standard numerical methods are not applicable.

DOI: 10.1134/S2635167624602638

INTRODUCTION

Sources of hard synchrotron radiation (**SR**) of the third and fourth generations are promising tools for nanotechnology, in particular nanophotonics. Development of the theory of focusing hard SR beams is of fundamental importance for a correct understanding of the possibilities of creating beams with a transverse size on the order of several nanometers. Currently, there are fundamentally different methods for focusing SR beams. The most attractive focusing method is based on the refraction of rays at the boundary of two media. However, such a method remained unimplemented for a hundred years after the discovery of X-rays, since this radiation, including SR, interacts very weakly with matter and is therefore only weakly refracted.

The first time that a SR beam was focused in this way was in 1996 [1] using a compound refractive lens (**CRL**). The idea was to use a periodic structure in which each period contains a refractive element (Fig. 1) with a relatively large radius of curvature of concave parabolic surfaces, which can be manufactured using modern technologies. The refraction effect is enhanced by the multiple repetition of such elements. As a result, a new focusing device appeared, which had its own periodic structure, and it became necessary to develop a theory of the propagation of SR through such a structure.

The most universal way to calculate such a structure involves sequential calculation of the propagation of the radiation wave function (**RWF**) through each element by examining it in the approximation of a thin lens. In this approximation, the length of the lens is only taken into account in the phase of the transmission function, and the distance to the next lens is considered free space. This calculation method is very expensive if the CRL has several hundred periods. In addition to the inconvenience associated with long calculation times, rounding errors can accumulate. A simpler analytical theory is needed that allows one to obtain information about the properties of CRLs in physically understandable terms. It was proposed in 2002 [2, 3]. In particular, the propagator of a long CRL, i.e., the wave function of the radiation at the output of the CRL for a point source at its input, was analytically calculated. The analytical formula for the propagator allows us to solve the problem in the general case, i.e., for an arbitrary RWF at the input to the CRL, in the form of an integral of the convolution of the RWF at the input with the propagator. As a result, we obtain the RWF at the output of the CRL and its further distribution is calculated using standard methods.

However, the calculated propagator has one drawback: it does not take into account the real CRL aperture. Initially this did not create any problems. The point is that the first CRLs had a very large aperture and the beam inside the CRL was limited due to absorption. Therefore, the actual aperture did not influence the result, but everything was determined by the effective aperture, i.e., that part of the actual aper-

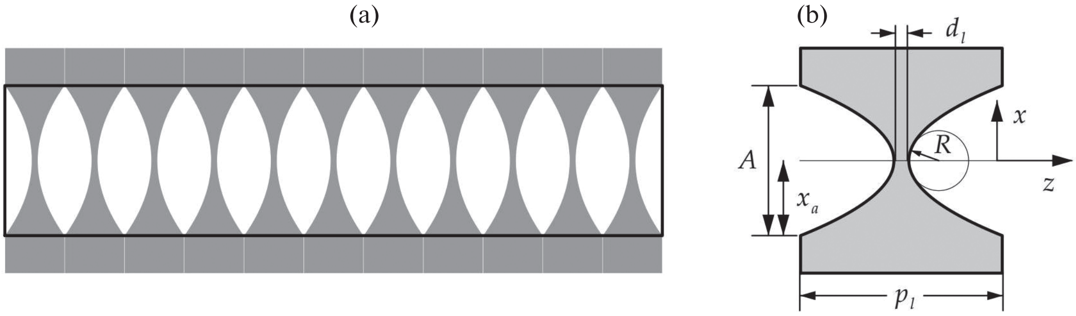

Fig. 1. General view of a compound refractive lens as a periodic structure (a). The parameters of one period, where A is the aperture, $x_a = A/2$, R is the radius of curvature of a parabolic surface, d_l is the thickness of the thin part, p_l is the length of the period, and the coordinate axes (b).

ture within which the RWF remained nonzero at the output of the CRL. All other radiation was absorbed inside the CRL. In cases where the effective aperture is smaller than the real aperture, calculations using the propagator give the correct answer, and in a large number of cases this answer can be obtained in analytical form. Many papers have been published [4, 5] that discuss some of the features of focusing SR beams with long CRL, in particular the coherence length, rocking curve, image of objects, etc.

One of the most interesting questions is how small a beam can be focused using a long CRL. In [6] a general statement was made that there is no way to make the size of the focused beam smaller than w_c, which is determined by the formula $w_c = \lambda(8\delta)^{-1/2}$, where λ is the wavelength of monochromatic radiation, $\delta = 1 - n$ is the refractive index and n is the absolute refractive index. However, according to the developed theory, if the CRL has an effective aperture determined by absorption, then it gives a higher transverse size of the focused beam. To obtain a smaller size, it is necessary to decrease the aperture and increase the photon energy, which will reduce the effect of absorption. Currently, this can only be done with planar (nanofocusing) CRLs, which make focusing one dimensional, i.e., only along the axis x. In this case, the theory, which does not take into account the real aperture, becomes inapplicable.

A more advanced theory is needed that takes this aperture into account. Attempts to partially take into account the real aperture were made in [4, 5], however, not entirely correctly and only in the case of an incident plane wave, more precisely at a large distance from the source to the CRL and with optimal orientation of the CRL axis, when the point source is located on this axis. In this paper, this problem is solved for the first time in the general case of a point source located at any distance and at any shift from the CRL axis. Such a calculation can be performed within the framework of new geometrical optics, consistent with the propagator, and with the correct consideration of partial absorption, which is also derived from the propagator.

TAKING INTO ACCOUNT THE LONG CRL APERTURE

Figure 1a shows a general view of a long CRL and highlights the focusing region, which has a height equal to the aperture A, and length $L = p_l n_l$, where p_l is the length of one period and n_l is the number of periods. Figure 1b shows the main parameters of one period, including the radius of curvature of concave parabolic surfaces R and the thickness of the thinnest part d_l in the middle of the period. At the same time $p_l = x_a^2/R + d_l$, where $x_a = A/2$. The coordinate axes used in the calculations are also shown. In the analytical theory [2, 3] the variable thickness of the material is replaced by a variable density, and the material along the axis z is considered to be continuous with uniform density. This approximation is the opposite of the calculation method described in the Introduction, when the lens material, on the contrary, is compressed into a line with variable density, and the entire space of the period is empty. Having two opposite approximations makes it easy to evaluate the accuracy of calculations under the condition that both approximations give the same answer with small deviations. This is indeed almost always the case.

In [4] it is shown that the ray trajectory inside a long CRL is described by the equation

$$x(z) = x_0 \cos(z/L_c) + \theta_0 L_c \sin(z/L_c), \quad L_c = (p_l R/2\delta)^{1/2}. \tag{1}$$

where x_0 and θ_0 is initial coordinate and angle of the beam, i.e., when entering the CRL, in the case where $z = 0$. There is also an equation for the angle

$$\theta(z) = dx/dz = \theta_0 \cos(z/L_c) - (x_0/L_c)\sin(z/L_c). \tag{2}$$

Let a SR point source be located at a distance z_s from the origin of the CRL and shifted from the axis z at a distance of x_s. Then $\theta_0 = (x_0 - x_s)/z_s$ and only the coordinate remains arbitrary x_0 provided that $|x_0| < x_a$. Obviously, the ray trajectories inside the CRL must not go beyond the aperture, i.e., the condition $|x(z)| < x_a$ must be satisfied in the entire interval $0 < z < L$.

Three cases should be distinguished. The first is when the source coordinate x_s is within the CRL aperture size, i.e., $|x_s| < x_a$. Then the focused rays inside the lens first diverge and it is necessary to ensure that the turning points of the divergence to convergence are located inside the CRL. If this is not the case, the beam will extend beyond the CRL aperture. It is enough to put a slit equal to the aperture of the CRL at the end of the CRL, and such a beam will not propagate further. In this case, from Eq. (1) we determine the interval of acceptable values of the coordinate x_0. Substituting into (1) the dependence θ_0 on the position of the source, we obtain the equation

$$x(z) = x_0 \cos(z/L_c) + (x_0 - x_s)(L_c/z_s)\sin(z/L_c). \quad (3)$$

It is necessary to determine the minimum and maximum points of this function. As is known, at these points the derivative with respect to z equals 0. Accordingly, we obtain an equation from which we determine the coordinate z, where the rays turn from divergence to convergence,

$$z = L_c \arctan(\underline{u}_s (x_0 - x_s)/x_0), \quad u_s = L_c/z_s. \quad (4)$$

We substitute this value into (3) and determine the following values x_0, at which $x(z) = \pm x_a$. Then we substitute them into (4) and make sure that the value z is in the range from 0 to L. It is obvious that always $z > 0$, but it may be that $z > L$. Then the CRL will not focus. It is convenient to enter the variable x from the condition $x_0 = x_s + x$ and consider the functions

$$F_{u,d}(x) = \pm x_a - (x_s + x)\cos(a) - (xu_s)\sin(a), \quad a = \arctan(u_s x/(x_s + x)). \quad (5)$$

Then we determine the points where these functions are equal to zero within the interval from $-(x_a + x_s)$ to $x_a - x_s$. Accordingly, we obtain the input points $x_{0u,d}$ and we easily find the output points $x_{1u,d}$ from (3) at $z = L$.

In the second case, when $x_s > x_a$, the upper point is known exactly. She is equal $x_{0u} = x_a$. The angle is also known exactly $\theta_{0u} = (x_a - x_s)/z_s$ and it is negative. It is necessary to check that the output point $x_{1u} = x_{0u}C_L + \theta_{0u}L_cS_L$, where $C_L = \cos(u)$, $S_L = \sin(u)$, was greater than $-x_a$, i.e., the beam did not cross the edge of the aperture inside the CRL. From here on $u = L/L_c$. If there is a slit beyond the end of the lens, then such a ray will not propagate. For the lower point the condition remains as in the first case. The third case is the opposite of the second, i.e., it is implemented when $x_s < -x_a$. Here, the upper and lower parts simply change places.

Such an analysis allows, in particular, to determine the CRL rocking curve, i.e., the region of all possible values x_s for a given distance z_s. After the two pairs of values $x_{0u,d}$ and $\theta_{0u,d}$ have been determined, we can easily calculate the two pairs of values $x_{1u,d}$ and $\theta_{1u,d}$ from (1) and (2) at $z = L$. The source image using the long CRL is located at the intersection of two rays defined by these pairs of values. To do this, we solve the equation

$$x_{1u} + \theta_{1u}z = x_{1d} + \theta_{1d}z, \quad (6)$$

from which we find the focal length z_f, measured from the end of the CRL, and the shift of the source image point x_f from the axis z

$$z_f = A_1/(\theta_{1d} - \theta_{1u}), \quad A_1 = x_{1u} - x_{1d}, \quad x_f = x_{1u} + \theta_{1u}z_f. \quad (7)$$

In the case of a short CRL, when $L \ll L_c$, the given equations give a known result. At $u \ll 1$ you can use the approximation $\cos(u) = 1$, $\sin(u) = u$. Moreover, one can use the approximation $x_{0u,d} = \pm x_a$. In the same approximation $x_{1u,d} = \pm A/2$. That is, the output aperture does not change. However, the angles change and the second term here has a value no less than the first, i.e.,

$$\theta_{1d,u} = \pm x_a \left(L/L_c^2 - 1/z_s\right) - x_s/z_s. \quad (8)$$

Substituting this into (9), we obtain

$$z_f = (Z_f^{-1} - z_s^{-1})^{-1}, \quad x_f = -x_s z_f/z_s, \quad Z_f = L_c^2/L = R/(2n_l\delta). \quad (9)$$

These formulas are widely used in short CRL focusing calculations.

So, geometrical optics gives very useful information, namely the limits of the region (x_{1d}, x_{1u}), from which the RWF is focused during further propagation into free space. Also known are the angles $\theta_{1u,d}$, which these rays form with the z axis. They give the derivative of the phase with respect to x according to the well-known rule $d\varphi/dx = (2\pi/\lambda)\theta$. Using this rule, it is easy to determine the RWF at the output of the CRL in the case where absorption is zero:

$$\psi(x) = (A_0/A_1)^{1/2} \exp(i\varphi(x))\theta(x_{1a} - |x - x_{1c}|), \quad (10)$$

$$\varphi(x) = a\left(2x_f x - x^2\right), \quad A_0 = x_{0u} - x_{0d}, \quad a = \pi/(\lambda z_f). \quad (11)$$

Here $x_{1a} = A_1/2$, $x_{1c} = (x_{1u} + x_{1d})/2$, $\theta(x)$ is a Heaviside function equal to zero if $x < 0$, and one if $x > 0$, and the parameters z_f and x_f are defined in (7). The phase is written without taking into account the constant term, which does not affect the intensity. The factor before the exponent takes into account the density of the rays. If the RWF normalization is such that the intensity is unity at the input to the CRL, then at the output the integrated intensity cannot change unless there is absorption. Therefore, the RWF amplitude increases as the illuminated area decreases.

The RWF at a focal length z_f from the end of the CRL is determined by the convolution of function (10) with the Fresnel propagator:

$$\psi_f(x) = (A_0/A_1)^{1/2} C_0 \int dx_1 \exp\left(ia[x - x_1]^2\right) \psi(x_1),\quad C_0 = (i\lambda z_f)^{-1/2}. \tag{12}$$

The integral is calculated analytically and is equal to

$$\psi_f(x) = (A_0 A_1)^{1/2} C_0 C_p(x) F_0\left(aA_1[x - x_f]\right), \tag{13}$$

$$C_p(x) = \exp\left(ia\left[x^2 - 2(x - x_f)x_{1c}\right]\right),\quad F_0(u) = \sin(u)/u. \tag{14}$$

Function $F_0^2(u)$ is equal to 1/2 at $u = C_h$ and $C_h = 1.3915574$. Therefore, the half-width of the peak is $w_f = 2C_h\lambda z_f/\pi A_1 = 0.8859\ \lambda/\alpha$. Here $\alpha = A_1/z_f$ is the angular width of the beam that is focused. It is easy to verify that the integral in infinite limits of intensity $I_f(x) = |\psi_f(x)|^2$ is equal to A_0, i.e., the law of conservation of energy is fulfilled. The maximum intensity value is equal to $I_{fma} = I_f(x_f) = A_0A_1/\lambda z_f$. The product of I_{fma} is $w_f = 0.8859A_0$.

Interestingly, according to (7) $\alpha = \theta_{1d} - \theta_{1u}$, i.e., the angular width is actually defined as the difference between the maximum and minimum angles that the rays make with the z axis. The larger the angular width of the focused beam, the smaller the size of the beam at the focus. It does not matter where this angular width is formed, in the CRL itself or outside of it. The result does not depend on the beam width at the output of the CRL. Sometimes it is advantageous to make the CRL shorter with the same focusing result.

TAKING INTO ACCOUNT ABSORPTION IN A LONG CRL

The complete absence of absorption in a long CRL can only be achieved with very small apertures and very high photon energies for materials consisting of light atoms. In real situations, absorption is always present. Generally speaking, there are two ways to calculate the phase in geometrical optics. In a CRL the material gradually changes thickness parabolically while maintaining the same density. Locally, the phase changes only where there is matter. One could calculate the length of that part of the trajectory where the rays propagate through a substance and multiply it by a factor of $2\pi\delta/\lambda$. But absorption occurs along the same sections of the trajectory. Therefore, the total absorption must be proportional to the phase, for this reason the complex phase is easily obtained after multiplying it by $(1 - i\gamma)$, where $\gamma = \beta/\delta$. This is exactly what is done when calculating short CRLs.

However, numerical calculations show that this is not correct for long CRLs. Geometrical optics matched with a long CRL propagator correctly calculates the phase, but the absorption is different and larger than that given by the approach described above. Analysis of the results of numerical calculations showed that calculations based on the long CRL propagator do not take into account the aperture, but correctly give both the RWF phase and absorption. Calculation of the RWF at the end of a long CRL for a point source was performed in [3]. Using formulas (17) and (18) from this work, the answer can be written as an exponential function with complex coefficients:

$$\psi(x) = \exp\left(ia_0 + ia_1x + ia_2x^2\right), \tag{15}$$

$$a_0 = (\pi c_L/\lambda r_g)x_s^2 - 2\pi\eta d_l n_l/\lambda - i\log(z_s/r_g)/2,\quad \eta = \delta(1 - i\gamma), \tag{16}$$

$$a_1 = -2\pi x_s/\lambda r_g,\quad a_2 = (\pi/\lambda r_g)(c_L - s_L z_s/z_c),\quad r_g = z_s c_L + z_c s_L. \tag{17}$$

From here on, the complex parameters $z_c = L_c(1 - i\gamma)^{-1/2}$ and c_L, s_L are used. The latter differ from the parameters C_L, S_L replacement in the argument L_c on z_c.

The argument of the exponent in (15) also contains a term that does not depend on x. The real part of the coefficient a_0 gives a constant phase that does not affect the recorded intensity. However, its imaginary part does have an effect, and this is important. A term is also added to it that describes the absorption at thin parts of the period, which do not participate in the focusing process, but absorb SR. Interestingly, additional absorption also occurs when the source is not on the CRL axis. It can be shown that in the absence of absorption, RWF (14) coincides with RWF (10) up to a constant phase and a factor determining the aperture compression. It is this factor that cannot be obtained using the CRL propagator.

In [5] they tried to use a propagator only within the CRL, and to place a slit in front of the CRL, the size of which is equal to the size of the aperture. Moreover, the integral within the aperture was approximately calculated using the steady-state phase method in order to obtain an analytical solution. As it turns out, this solution is only suitable for a plane incident wave, since it does not take into account the phase of the incident radiation, although it does take into account the compression of the aperture approximately. It is possible to obtain the result in a more general form, i.e., for a point source with parameters z_s and x_s. This result should replace formula (16) of work [5]. Let us write it in a form similar to (14), namely

$$\psi(x, L) = \exp\left(ia_0 + ia_1x + ia_2x^2\right)A(x_1), \tag{18}$$

$$a_0 = -(\pi/x_L^2)(bx_s)^2/(c_L + b) - 2\pi\eta d_l n_l/\lambda + i\log(c_L + b)/2, \quad (19)$$
$$x_L = (\lambda z_c s_L)^{1/2},$$

$$a_1 = -(\pi/x_L^2)2bx_s/(c_L + b),$$
$$a_2 = (\pi/x_L^2)[c_L - 1/(c_L + b)), \quad b = x_L^2/\lambda z_s. \quad (20)$$

Although the coefficients in the argument of the exponent are written in a different form, their meaning will be explained below, they exactly coincide with those written in (16) and (17). We note that it is difficult to prove this for the coefficient a_2, but it is possible. The easiest way is to verify this numerically.

The main difference between formula (18) and formula (15) is that the result may depend on the amplitude of the RWF at the input, i.e., the function $A(x)$, which does not necessarily have to be equal to one. In (15) it was assumed that it was equal to unity, and the integral was calculated in infinite limits. In (18) the phase of the RWF from a point source is taken into account, and the amplitude can be arbitrary, in particular, it can limit the wave incident on the CRL by the aperture. It is important that for each point x there is such a point x_1, in which the amplitude must be taken into account. It is determined by the formula

$$x_1 = (x + bx_s)/(c_L + b). \quad (21)$$

For the amplitude in this formula, absorption can be neglected, i.e., it can be assumed that $\gamma = 0$. The argument of the exponent was also calculated at this point, therefore the form of the coefficients in (19) and (20) differs from (16) and (17). Formula (21) shows, in particular, how the aperture is transformed when the RWF propagates inside the CRL from the input to the output.

In the XRWP program, the calculation method based on formulas (15)–(17) is assigned number 1, and the calculation method based on formulas (18)–(20) is assigned number 3. While method 1 does not take into account the actual aperture, method 3 takes it into account relatively well if the source is on the CRL axis and the distance significantly exceeds the CRL length. But the most accurate calculation of the aperture is performed according to formulas (9)–(11). In this case, the RWF phase should actually be the same as in methods 1 and 3, and the main difference is the parameters A_0 and A_1, i.e., the size of the beam at the input to the CRL and at the output from the CRL. For convenience of calculations, we will write out the real and imaginary parts of the coefficients in the argument of the exponent in explicit form. Taking into account the smallness of the parameter γ, it is sufficient to calculate the imaginary part in the linear approximation of γ. Thus,

$$a_{2r} = -a, \quad a_{2i} = Ga_{2r},$$
$$G = -(\gamma/2)(C_1 + [uC_3 + C_2/S_L]/C_4), \quad (22)$$

$$a_{1r} = 2x_f a, \quad a_{1i} = -(\gamma/2)C_2 a_{1r}, \quad a_{0r} = 0, \quad (23)$$

$$a_{0i} = (\gamma/2)(\pi/\lambda)\left[(x_s/z_s)^2 L_c S_L (C_2 - C_1)/C_3 + 4\delta d_l n_l\right] + \log(C_3)/2, \quad (24)$$

$$C_1 = 1 - uC_L/S_L, \quad C_2 = (S_L(u + u_s) - C_L u u_s)/C_3,$$
$$C_3 = C_L + u_s S_L, \quad (25)$$

$$C_4 = S_L - u_s C_L, \quad z_f = L_c C_3/C_4,$$
$$x_f = -z_f (x_s/z_s)/C_3. \quad (26)$$

Parameter a is defined in (11). The calculation method using formulas (10), (11), and (22)–(26) in the XRWP program is assigned the number 5. This is a new version of the theory.

We note that parameters such as z_f, x_f, and $A_1/A_0 = C_3$, do not depend on the aperture. For this reason, formulas (25) and (26) exist for them, and their calculation by the coordinates and angles of the rays precisely coincides with the calculation using these formulas. That is, the geometrical optics used is fully consistent with a long CRL propagator. The radiation wave function at the focal length is now determined by a more complicated integral, which we rewrite as

$$\psi_f(x) = C_0\int dx_1 \exp\left(ia[x - x_1]^2\right)\psi_1(x_1)\psi_2(x_1), \quad (27)$$

$$\psi_1(x) = \exp\left(ia\left[2x_f x - x^2\right]\right)\theta(x_{1a} - |x - x_{1c}|), \quad (28)$$

$$\psi_2(x) = \exp\left(-a_{0i} - a_{1i}x - a_{2i}x^2\right). \quad (29)$$

Parameter C_0 is defined in (12). Unlike (12), the integral in (27) is not calculated analytically. It can be transformed into the form

$$\psi_f(x) = (A_1/2)C_0 C_p(x)\psi_2(x_{1c}) \times \int du E(x,u)\theta(1 - |u|), \quad (30)$$

$$E(x,u) = \exp\left(-iaA_1[x - x_f]u - hu - gu^2\right), \quad (31)$$

$$h = A_1(a_{1i}/2 + a_{2i}x_{1c}), \quad g = a_{2i}A_1^2/4. \quad (32)$$

If there is no absorption, then taking into account that $\psi_2(0) = (A_0/A_1)^{1/2}$, $h = g = 0$, we obtain formula (13).

A simpler case with absorption is obtained when $x_s = 0$. In this case $x_f = 0$, i.e., $a_{1r} = a_{1i} = x_{1c} = 0$, and the integral is transformed to the form

$$\int du E(x,u)\theta(1 - |u|) = 2F(u_0), \quad u_0 = aA_1 x, \quad (33)$$

$$F(u_0) = \int du \cos(u_0 u)\exp\left(-gu^2\right)\theta(u[1 - u]). \quad (34)$$

Function $F(u_0)$ was studied in detail in [4, 8]. The half width $w(g)$ of the curve $F^2(u_0)$ defines the half width w_f of the SR beam at the focus. The following approximations were obtained: $w(g) = 2.783 + 0.498\,g$ in the range of values g from 0 to 6 and $w(g) = 2.355\,g^{1/2}$

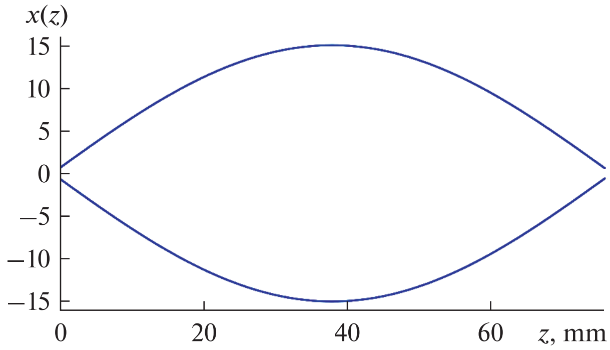

Fig. 2. Ray trajectories limiting the focused part of the synchrotron radiation beam for a lens with parameters: A = 30 μm, p_l = 62 μm, d_l = 2 μm, R = 3.75 μm, n_l = 1248, and photon energy of 50 keV.

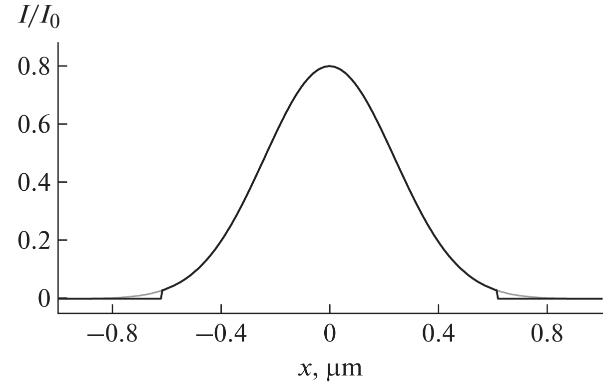

Fig. 3. Distribution of the relative radiation intensity on the output face of the lens for the parameters specified in the caption to Fig. 2, calculated by two methods of the XRWP program, namely method 5 (black curve) and method 3 (gray curve).

for $g > 6$. That is, it turns out that the presence of absorption at a given aperture size leads to an increase in the transverse size of the SR beam at the focus. Decreasing the aperture also increases the size, because

$$w_f = w(g)/(aA_1). \tag{35}$$

CALCULATION RESULTS

Let's consider an example of applying the new version of the theory. Let the CRL have the following parameters: A = 30 μm, p_l = 62 μm, d_l = 2 μm, R = 3.75 μm. An experiment with such a CRL was first presented in [9]. Let the photon energy be 50 keV (λ = 0.0248 nm), then L_c = 2.4637 cm. The most exotic case is the one in which the CRL focuses a point source at its end, which is located at its beginning. This is possible with $L = \pi L_c$ = 77.399 mm. That is, the specified CRL must have n_l = 1248 periods, which gives the value L = 77.376 mm, which is the closest match to what is needed. But this is not very convenient for analysis, so let's consider a slightly simpler case, when z_s = 1 mm and n_l = 1216. At the same time L = 75.392 mm and z_f = 1.0093 mm.

The trajectory of the rays limiting the beam in the CRL is shown in Fig. 2. The beam has its maximum size, equal to the CRL aperture, in the middle, as follows from the symmetry of the problem. It is easy to see that at the beginning and at the end of the trajectory the lines are almost straight, i.e., in the middle of the CRL aperture refraction of the rays is very weak. A strong change in trajectory occurs only at the edges of the aperture, where SR is absorbed most strongly. It is obvious that the left half of such a CRL transforms the diverging radiation from a point source near the front of the CRL into a parallel beam, while the right half, on the contrary, focuses the parallel beam near its rear part. In both cases, the number of periods is 2 times less, but still large.

Figure 3 shows the intensity distribution, i.e., the square of the absolute value of the RWF, on the output face of the CRL. All calculations were made using the XRWP program [7]. The calculation is performed by two methods, namely method 5 (black curve), first presented in this paper, and method 3 (gray curve), which was described in [5], but only for a plane incident wave, and is presented here in the more general case of a point source. In method 3, the CRL aperture is not taken into account exactly, but can be formally taken into account by correctly choosing the slit before entering the CRL if the source is located on the CRL axis. But in this case the slit was 2 microns, which is more than what would be expected from calculations. For this reason, the grey curve differs from the black curve at the ends, i.e., beyond the beam size that is accurately determined in Method 5. Interestingly, in this case the results for methods 1 and 3 coincide, since the beam is completely absorbed at a slit width of 2 μm.

Absorption in method 5 is used in a linear approximation with respect to the parameter γ for the imaginary parts of the coefficients in the argument of the exponent. But numerically, the curve of method 5 almost completely coincides with the curve of method 3 at the center of the peak. As for methods 2 and 4, which calculate the convolution integral with the propagator, they do not give correct results in this particular case. One of the reasons is that the input RWF oscillates very strongly, and the numerical fast Fourier transform method [10] is not able to give the correct answer. We note that the large number of iterations in method 2 and the very complex form of the propagator in method 4 also create problems. Therefore, for the case under consideration, only analytical methods are applicable.

The beam intensity at the output of the CRL is normalized to the intensity at the input. Since the beam size at the output is the same as that at the input, the difference is only due to a decrease in intensity due to absorption on thin parts of the material in the middle of the periods, i.e., when multiplied by the coefficient $\exp(-4\pi\beta d_l n_l/\lambda)$. The beam half width at the focus can be easily calculated using formula (35) using the values of the parameters $A_1 = 1.228$ μm and $g = 1.654$. As a result we get $w_f = 23.4$ nm. Parameter w_c, which is indicated in the introduction, in this case is equal to 20.0 nm. It can be seen how the absorption in CRL results in a result above the previously set limit.

CONCLUSIONS

For the first time, an adequate method for taking into account the aperture of long CRLs has been developed based on geometrical optics consistent with a long CRL propagator, which was obtained in [2, 3] more than 20 years ago. The aperture calculation method is not direct, but all calculations are performed quite quickly. Analytical formulas for the RWF at the output of the CRL in the case of SR from a point source are obtained, taking into account the shift from the CRL axis, which allows calculating the rocking curve of long CRLs more accurately than was done previously.

The numerical example considered is illustrative. It does not cover the full range of issues related to SR nanofocusing using a CRL. At the same time, it shows the advantages of analytical theories before numerical methods, which cannot always be performed in standard ways. Another great advantage of analytical formulas is the ability to quickly and easily select interesting and useful options for implementing the desired effects.

A more detailed analysis of focusing using both long CRLs at high energies and cascade CRLs where the aperture is variable will be performed in the future. It is important that a method has been found to accurately take into account the aperture of long CRLs using fairly simple means.

FUNDING

The work was carried out within the framework of the state assignment of the National Research Center "Kurchatov Institute."

CONFLICT OF INTEREST

As author of this work, I declare that I have no conflicts of interest.

REFERENCES

1. A. Snigirev, V. Kohn, I. Snigireva, et al., Nature **384**, 49 (1996). https://doi.org/10.1038/384049a0
2. V. G. Kohn, JETP Lett. **76**, 600 (2002). https://doi.org/10.1134/1.1541043
3. V. G. Kohn, JETP **97**, 204 (2003). https://doi.org/10.1134/1.1600812
4. V. G. Kohn and M. S. Folomeshkin, J. Synchrotron. Rad. **28**, 419 (2021). https://doi.org/10.1107/S1600577520016495
5. V. G. Kohn, J. Synchrotron Rad. **29**, 615 (2022). https://doi.org/10.1107/S1600577522001345
6. C. Bergemann, H. Keymeulen, and J. F. van der Veen, Phys. Rev. Lett. **91**, 204801 (2003). https://doi.org/10.1103/PhysRevLett.91.204801
7. V. G. Kohn, XRWP computer program (2024). https://xray-optics.ucoz.ru/XR/xrwp.htm
8. V. G. Kohn and M. S. Folomeshkin, Nanobiotechnol. Rep. **17**, 126 (2022). https://doi.org/10.1134/S2635167622010086
9. A. Snigirev, I. Snigireva, M. Lyubomirskiy, et al., Opt. Express **22**, 25842 (2014). https://doi.org/10.1117/12.2061616
10. W. Cooley and J. W. Tukey, Math. Com. **19**, 297 (1965).

Publisher's Note. Pleiades Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.
AI tools may have been used in the translation or editing of this article.

SPELL: OK