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DIFFRACTION AND SCATTERING OF IONIZING RADIATIONS

Accurate Measurement of the Rocking Curve of a Planar Compound Refractive Lens for Synchrotron Radiation Focusing

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Abstract—The results of the first measurement of the rocking curve of a nanofocusing compound refractive lens made of silicon, used for focusing synchrotron radiation (SR) at the "KISI-Kurchatov" source, are presented. The obtained curve is accurately approximated by a Gaussian function, and its width is in agreement with the previously developed analytical theory, describing the SR propagation in multi-element focusing systems. The results demonstrate the feasibility of using the rocking curve as an alignment characteristic of the experimental setup when working with silicon lenses at second-generation SR sources.

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INTRODUCTION

Compound refractive lenses (CRLs) [1] are one of the main elements of the infrastructure of third- and fourth-generation synchrotron radiation (SR) sources [2]. These lenses do not deviate the SR beam, improve its coherent properties, and efficiently remove heat. Their operation is reliable and predictable because of the existence of a simple and efficient theory of SR beam transmission through a CRL. For short CRLs, whose length is several times smaller than the focal length, standard geometric optics works well. For long CRLs, which do not satisfy the aforementioned condition, a more complex theory was developed based on the CRL propagator, which was analytically calculated for the first time in [3, 4] in 2002.

An analytical theory based on recurrence formulas was developed later [5, 6]. It was shown that, when the effective CRL aperture [7] is smaller than the real aperture and the dependence of the wave function (WF) of the incident radiation on the transverse coordinate is described by an exponential of a quadratic trinomial with three complex coefficients, further propagation of this wave through a system of parabolic lenses and through air does not change the analytical form of the WF. Only the coefficients change (the new coefficients are obtained from the old ones using analytical formulas), and the radiation intensity is described by a Gaussian function.

This approach makes it possible to derive analytical formulas for all parameters characterizing an SR beam during its transmission through a system of CRLs. The main parameters are the focal length z_f; the SR beam size in the focus, w_f; and the SR beam angular divergence after the focus, a_f. The parameters that are less critical but also have practical importance are the focal depth l_f, i.e., the distance at which the focal size is approximately preserved during the motion along the optical axis, and the width of the CRL rocking curve, a_r, i.e., the angular range within which the CRL transmits the SR beam.

The rocking curve of a long CRL was experimentally measured for the first time in [8]. The corresponding theory had not been developed by that time, and the only conclusion drawn was as follows: the measured parameter a_r turned out to be two times larger than the value following from the simple geometric optics. Later, a more accurate value was obtained for the rocking curve width in [9] in the form of an analytical formula, taking into account the dependence on the experimental parameters and source sizes. It was also shown that the rocking curve is a Gaussian function, and that the value of the RC width measured in [8] is in complete agreement with the theory. In the first (and the only until now) experiment, a two-dimensional focusing CRL, consisting of elements having a round aperture and a rotational parabolic shape of a refracting surface, was applied. That CRL was made of aluminum, had 407 elements with a curvature radius of 200 μm at the parabola apex and a length of 1 mm. The total lens length, even without

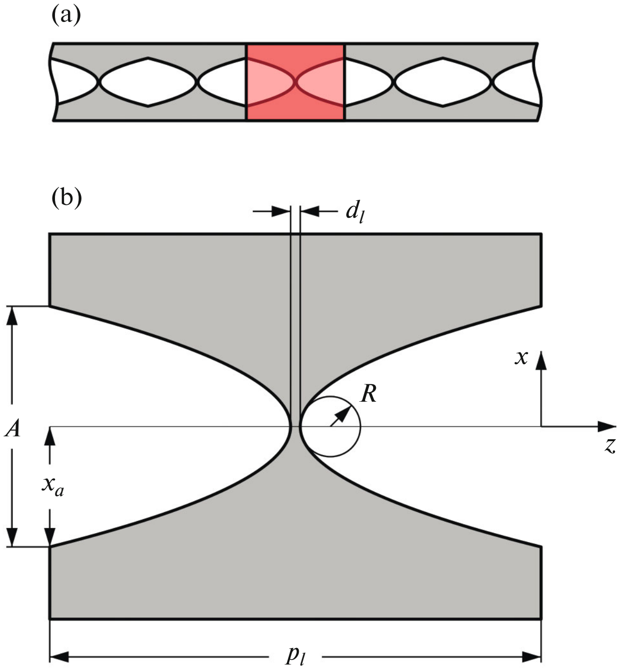

Fig. 1. (a) General view of a compound refractive lens as a periodic structure. (b) Parameters of one period: (A) aperture, $x_a = A/2$, (R) curvature radius of the parabolic surface, (d_l) thickness of the thin part, (p_l) period length, and coordinate axes.

housing, was 40.7 cm. The experiment was performed on the third-generation source ESRF (Grenoble, France).

In recent years, planar nanofocusing silicon CRLs, fabricated using electron lithography and anisotropic deep plasma etching, have been increasingly used [10, 11]. Planar CRLs consist of elements whose refracting surface has a shape of a parabolic cylinder; thus, they perform one-dimensional focusing. These compact lenses have a small aperture and provide a high degree of coherence [9], due to which they can be efficiently used even on the second-generation sources, such as “KISI-Kurchatov” [12–14]. In this paper, we report the results of the first measurement of the rocking curve of a nanofocusing CRL with an aperture of 50 μm. It is shown that, to obtain accurate results, it is necessary to record a two-dimensional beam pattern and select the part directly corresponding to the CRL focusing. In this case, one obtains a curve in the form of a Gaussian function with a width consistent with the theory.

THEORY

Figure 1 shows a general view of a CRL and the parameters of one element. Let us assume that the SR beam width at the CRL output is much smaller than the geometric aperture and does not depend on it, so that the radiation is completely absorbed in the thick part of the CRL far from the optical axis z. In this case, the transmission function of one CRL element can be written as

$$T(x, f_c) = \exp(-iK[\delta - i\beta]x^2/R) = \exp(-i\pi x^2/\lambda f_c), \quad (1)$$

where $\delta - i\beta = 1 - n$, n is the complex refractive index of the SR for a photon energy E, $\gamma = \beta/\delta$, $f_c = f/(1 - i\gamma)$, $f = R/2\delta$, R is the curvature radius at the parabola apex, $\lambda = hc/E$, $K = 2\pi/\lambda$, h is Planck’s constant, and c is the speed of light in vacuum. According to the geometric optics, if δ and β are much smaller than unity, the SR WF is simply multiplied by function (1) when passing through one lens element.

If the WF has a form of an exponential of a quadratic trinomial, only the coefficient of x^2 will change in it. It can be shown that the same occurs when calculating the convolution of the WF and Fresnel propagator, i.e., when the WF propagation through empty space is calculated. This can easily be understood in the framework of the following considerations. The calculation of a convolution is equivalent to the calculation of the product of the functions Fourier transforms. But the Fourier transform of an exponential of a quadratic trinomial remains the same exponential, only with other coefficients. To derive analytical formulas for converting the coefficients, it is necessary to perform relatively complex analytical calculations. This was done for the first time in [5] and then demonstrated again in [6]. These formulas are given below, with their derivation omitted.

Thus, let the WF have the following form after passing through k thin lenses and the distance between them:

$$\psi_k(x) = T(x, a_k)P(x - x_0, b_k)T(x_0, c_k), \quad (2)$$

where x_0 is the parameter of deviation of a point source from the optical axis. It is determined by the boundary condition and set in the beginning of the calculation, i.e., in the function $\psi_0(x)$. The function $P(x, z) = (i\lambda z)^{-1/2}\exp(i\pi x^2/\lambda z)$ is the Fresnel propagator.

After passing through another thin lens with a parameter f_c and the distance z in empty space, the WF has the same form (2) but with the index $k + 1$. The new parameters are calculated from the old ones using the formulas

$$(a_{k+1})^{-1} = b_k h, \quad b_{k+1} = z + b_k(1 - zg), \quad (c_{k+1})^{-1} = (c_k)^{-1} + zh, \quad (3)$$

where $g = (a_k)^{-1} + (f_c)^{-1}$, $h = g(b_{k+1})^{-1}$. The calculation sequence is as follows: g is calculated first, then b_{k+1}, then h, and then all other parameters. Transmission function (1) is applied at the center of each CRL element (Fig. 1), and the distance between closely packed elements is equal to their length. This is the essence of the compressed-lens approximation. It is fairly accurate in the regions where the lenses are thin, whereas in the regions of large lens thickness the radiation is absorbed and does not affect the result. To calculate a rocking curve, it is sufficient to know the WF at the lens end, because one has to compute the total intensity, which does not change in empty space.

After determining the parameters a, b, and c at the CRL end, one has to calculate the squared modulus of function (2) and integrate it over the coordinate x. The answer $I(x_0)$ is obtained in the form of a Gaussian function of the coordinate x_0. Specifically this is the rocking curve for a point SR source. Its argument is not quite correct, because, if a source is displaced by a coordinate x_0, the radiation from it is incident on the lens at an angle $\alpha = x_0/z_0$, where z_0 is the distance from the source to the CRL. To take into account the source size, one must make another step: calculate the convolution of the function $I(x_0 + x_s)$ and the source brightness function, $B(x_0)$. Here, x_s is the coordinate of the displacement of the SR source center from the optical axis. The function $B(x_0)$ is also a Gaussian function, so that shape of the curve will not change, but the width will change. As a result, we obtain the dependence $I_t(\alpha)$, where the argument is defined as $\alpha = x_s/z_0$.

The full width at half maximum (FWHM) of the function $I_t(\alpha)$ can be expressed analytically in terms of the parameters a, b, and c. It is equal to $a_r = e\sigma_0(\sigma_0^2 + \sigma_s^2)^{1/2}/z_0$, where $e = (8\ln2)^{1/2} = 2.355$, $\sigma_s = w_s/e$, w_s is the source size, $\sigma_0 = (2K[C - AM])^{-1/2}$, $M = B/(A - B)$, $A = -\mathrm{Im}(1/a)$, $B = -\mathrm{Im}(1/b)$, $C = -\mathrm{Im}(1/c)$. The parameter a_r can be calculated using the on-line program [15], written in the JavaScript language. The program very rapidly calculates the SR beam parameters after passing through one or several CRLs, with allowance for the experimental scheme parameters.

EXPERIMENTAL

The experiment was performed on the XCPM (X-ray Crystallography and Physical Materials Science) beamline of the "KISI-Kurchatov" source. A schematic of the experiment is presented in Fig. 2.

The SR generation at the XCPM beamline is performed by a bending magnet, installed at a distance $z_0 = 15$ m from the sample position on the five-circle goniometer (Huber). The SR source, formed by the bending magnet, has a shape elongated in the horizontal direction; it is approximated quite accurately by a two-dimensional Gaussian function with a half-width of ~100 × 1000 μm^2 in the vertical and horizontal directions, respectively. A double-crystal Si(111) monochromator (FMB Oxford) with a relative spectral resolution $E/E \sim 10^{-4}$ is used for SR beam monochromatization; it is not shown in Fig. 2. A more detailed description of the XCPM technical equipment can be found in [16].

SR focusing was carried out using a planar CRL [10, 11] on the surface of a single-crystal silicon plate

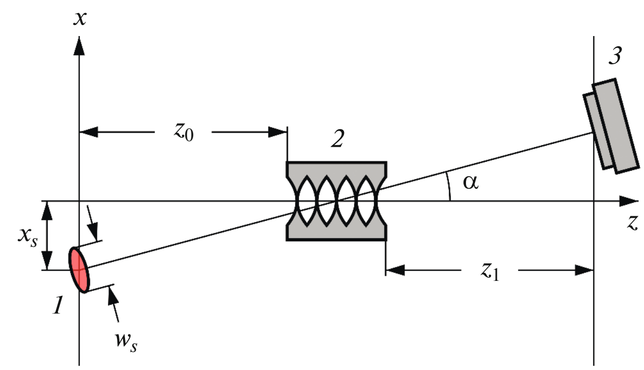

Fig. 2. Experimental scheme for measuring the CRL rocking curve: (*1*) extended SR source, (*2*) planar silicon CRL, and (*3*) area detector.

with an etching depth of 50 μm and the following period parameters: $A = 50$ μm, $R = 6.25$ μm, $d_l = 2$ μm, and $p_l = 102$ μm (Fig. 1). The total number of periods was $N = 196$, i.e., the CRL was long in the sense that its length exceeded several times the focal length. Planar CRLs perform one-dimensional focusing; i.e., they compress the SR beam in one plane. In the perpendicular plane, the radiation propagates without changing its direction. As a result, the focal spot is a narrow line with a longitudinal size equal to the etching depth, and the transverse size corresponds to the beam size at the focus. The CRL was installed on the goniometer so that focusing occurred in the vertical (x, z) plane, corresponding to the smallest SR source size.

The monochromator was tuned to the photon energy $E = 18$ keV. The theoretical focal length, counted from the CRL end, for the chosen energy and the aforementioned parameters of the experiment was 3 mm. The monochromatic SR beam before the CRL was limited in the vertical and horizontal directions using a pair of collimating slits (are not shown in Fig. 2) with a size of 50 μm, which corresponded to the CRL aperture A and the silicon etching depth. Before the measurements, the spatial and angular CRL position was aligned along the SR beam propagation direction. This position corresponded to zero displacement of the coordinate x_s of the SR source center from the optical axis z. Then the CRL was rotated by an angle α from the initial position around the axis passing through the CRL center perpendicular to the (x, z) plane. The rotation angle α was varied in the range from −1.75 to 1.75 mrad with a step of 0.175 mrad. The angular range was chosen proceeding from the theoretical value of the rocking curve half-width $a_r = 1.179$ mrad for the aforementioned parameters. The two-dimensional intensity distribution after the CRL in the transverse plane, $I(x, y)$, was recorded for each angular position. Thus, 21 images of focused SR beam intensity were registered during the experiment for different CRL angular positions.

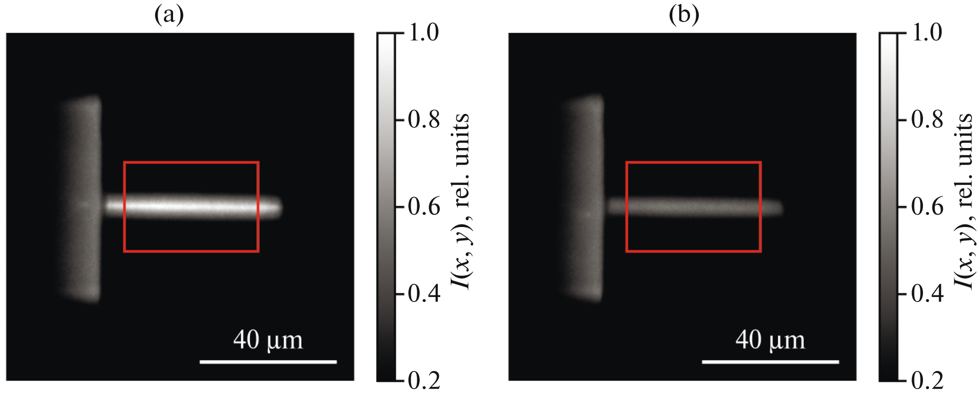

Fig. 3. Measured images of the SR intensity distribution after the CRL for the rotation angles α = (a) 0 and (b) 0.525 mrad.

The intensity distribution was recorded using a two-dimensional X-ray detector XSight Micron (Rigaku) based on a scintillation screen and an sCMOS sensor with an effective pixel size of 0.325 μm. The detector was located at a distance $z_1 = 10$ mm from the CRL end. The exposure time for each recorded two-dimensional image was 5 s.

Note that the experimental distance z_1 does not correspond to the CRL focal length; hence, the size of the recorded SR beam in the vertical direction was larger than the minimal one. However, since the rocking curve is an angular dependence of the total intensity, a slight broadening of the beam size is not a problem, because signal summation can be performed over any area of the recorded image.

RESULTS AND DISCUSSION

Figure 3 presents the measured SR beam images after the CRL for the rotation angles $\alpha = 0$ and 0.525 mrad. A focused linear SR beam transmitted through the CRL is observed at the image center. The beam size along the horizontal corresponds to the CRL structure etching depth in silicon and is approximately 50 μm. It can be seen that an increase in the CRL rotation angle expectedly leads to a decrease in the focused beam intensity. At the same time, the vertical beam position barely changes, because the CRL focuses radiation almost at its end, while the rotation angle is small.

In the left part of the images one can see a part of the SR beam that passed by the CRL along the silicon wafer surface, despite the use of a 50-μm collimating slit in the horizontal direction. This is due to the relatively large horizontal size of the SR source, because the radiation from the points shifted from the optical axis in the horizontal direction propagates at some angle to the silicon wafer surface. This effect is observed even for a slit with a significantly reduced opening size.

To exclude this background signal, the image corresponding directly to the focused SR beam was selected. This area is framed in Fig. 3. Then the signal was integrated over the selected area for all measured images, as a result of which experimental rocking curve $I_t(\alpha)$ was obtained. Note that a change in the range of integration (provided that the focal spot is captured) did not lead to any significant change in the shape of the $I_t(\alpha)$ curve and its FWHM.

The experimental dependence $I_t(\alpha)$ is presented in Fig. 4 (circles). The error in measuring the total intensity $I_t(\alpha)$ at each point is smaller than the circle size. The result of fitting the experimental data by a Gaussian with application of the least-squares method is also shown in Fig. 4. It can be seen that the experimental data are described with a high accuracy by a Gaussian function, which is consistent with the theory. The FWHM of the model curve after the fitting is 1.188 ± 0.021 mrad, which coincides (within the error) with the theoretical value of 1.179 mrad.

The CRL in the performed experiment is located so that the rotation axis passes through the lens center perpendicular to the drawing plane in Fig. 2. The silicon wafer surface is oriented parallel to the vertical shutters of collimating slits. If the aforementioned conditions are not fulfilled, angular rotation leads to displacement of the lens aperture center relative to the center of slits. As a result, the relative intensity of the forward SR beam transmitted above the silicon surface (Fig. 3) will change with a change in α.

This effect can be used to estimate the axial alignment of the experimental scheme when working with an integral detector, recording simultaneously the focused beam and the part of the radiation passed by the CRL. In the case of axis misalignment, the rocking curve recorded by this detector will differ from the theoretical one because of the changing contribution of the forward SR beam at the angular rotation of CRL. In the opposite case, the contribution of the radiation passed by the lens is the same for all angular points, and the rocking curve is consistent with the theory.

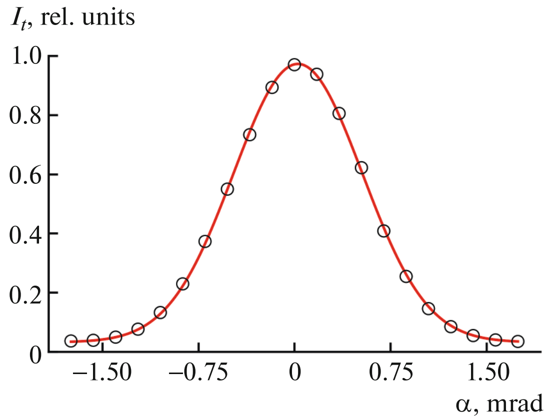

Fig. 4. Experimental CRL rocking curve (circles) in comparison with the fitting result (solid line).

For example, if the signal is integrated over the entire area of the images (Fig. 3) measured in the coaxial scheme, the rocking curve is also fitted with high accuracy by a Gaussian function with a FWHM of 1.18 ± 0.04, which corresponds (within the error) to the theoretical value.

CONCLUSIONS

The rocking curve of a long nanofocusing silicon lens was measured for the first time at the XCPM beamline of the "KISI-Kurchatov" source. An analysis of the obtained experimental data showed their complete correspondence with the previously developed analytical theory of focusing synchrotron radiation by multi-element lens systems. In particular, the recorded rocking curve is fitted with high accuracy by a Gaussian function with a FWHM corresponding to the theoretical calculation.

In addition, it is shown that, when one works with silicon lenses on second-generation SR sources using integral detectors, axial alignment of the experimental scheme must be provided to exclude the influence of the beam passed by the lens. The lens rocking curve can be used as a characteristic of the coaxiality of the experimental scheme alignment.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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